OFFICERS AND COUNCIL.

CONTENTS OF PROCEEDINGS.

NAME INDEX (ABSTRACTS).

the observed localization demonstrates the important result that the effect is due to a specific dynamical action of the waves, and not to mere general absorption of the radiation.

Let us consider the actual circumstances of the case. There are about 10^3 molecules of the sensitive medium in the length of a single wave of light : thus in the stationary wave-train all the parts of a single molecule would at any instant be moving with a sensibly uniform velocity, which increases and diminishes periodically. The vibration of the molecule would thus be, were it not for the influence of differences of inertia or elasticity between its parts and the surrounding æther, very nearly a swaying to and fro of it as a whole : if it were exactly this, it could not be expected to produce any breaking up of the molecule at all. Moreover, as at the antinodes of the vibration there is movement but no stress in the medium, so at the nodes there is stress but no movement; and it does not seem at all clear that alternating stress might not be as potent a factor in disintegration as alternating motion. A representation has been constructed by Lord Kelvin * of a system in which internal vibrations can be excited by simple translation, by means of the device of an outer shell imbedded in the æther and containing inside it masses with spring connexions ; and such a system might also be adjusted so as to respond to simple rotation, and therefore be excited at the nodes of the wave-train instead of the antinodes.

A theory based in this manner on difference of inertia must take the density of the æther to be very minute compared with that of matter ; therefore if the molecule is to have free periods of the same order of magnitude as the periods of the incident light-waves, the elastic forces acting between the atoms and concerned in these periods must be very intense. But Lord Kelvin's well-known estimate of the rigidity of the æther on this hypothesis makes it very small compared with the ordinary rigidity of material bodies †. In fact on Pouillet's data, which imply a considerable underestimate, the energy of the solar radiation near the sun's surface is about 4×10^{-5} ergs per cubic centimetre; it easily follows that

* "Lectures on Molecular Dynamics," Baltimore, 1884.
† *Cf.* Maxwell, *Encyc. Brit.*, article " Æther."

if the amplitude of the æthereal disturbance is, say, ϵ times the wave-length, the density of the æther must be about $1/10^{22}\epsilon$, and its rigidity, which is equal to the density multiplied by the square of the velocity of propagation, therefore $1/10\,\epsilon^2$. On an elastic solid theory it is desirable to have the density very small : thus if we adopt 10^{-2} as the maximum likely value of ϵ, the density of the æther comes out 10^{-18} of that of water, and its rigidity about 10^3, whereas the rigidity of steel or glass is of the order 10^{11}.

Now at first sight it would appear that the elastic tractions exerted by an æther of such small rigidity on an imbedded molecule swaying backwards and forwards in it, would be vanishingly small compared with the elastic forces between its constituent atoms which are concerned with free vibrations of the kind of period under consideration ; and that therefore they would be quite incompetent to produce violent disturbance in the molecule. But on a closer examination this difficulty may to a considerable extent be evaded.

Let us imagine an imbedded rigid nodule of linear dimension L, and let the force necessary to displace it in the æther in any manner through a distance x be Lx. Let us compare with it a similar nodule of linear dimensions κL displaced through a distance κx. There is complete dynamical similarity between the two cases ; the strains at corresponding points in the æther are equal, and therefore so are the tractions per unit area. Thus the forcive necessary in the latter case to produce the displacement κx is κ^2Lx, and therefore to produce the same displacement x as in the previous case a forcive κLx is required. If now instead of comparing the total forcives in the two cases we compare the forcives per unit volume, an increase of linear dimensions in the ratio of κ to one diminishes this forcive in the ratio of κ^2 to one. Thus, if only the atoms are taken small enough, an æther of very slight rigidity can exert a forcive on them which, estimated per unit volume, is of any order of magnitude we please. The features of the case are in fact analogous to those of the suspension of small bodies, such as motes, in a viscous fluid medium like the atmosphere : if only the particles are small enough they will float for an indefinitely great time against the force of gravity, even be they as

dense as platinum,—the only limit being in that case the one imposed by the molecular discreteness of the air itself.

It would appear that the application of this principle does much to vivify the notion of an elastic solid æther. A medium of this kind, which is excessively rare and, as a consequence, of very feeble elasticity, would exert practically negligible tractions on the surfaces of a mass of matter in bulk, while it may exert relatively very powerful ones on the individual atoms of which the mass is composed if only they are sufficiently small, it being of course supposed that the structure of the medium itself is absolutely continuous. And it would even appear that a medium of very small density and rigidity may be competent to excite powerful vibrations in the molecules notwithstanding the strength of the forcives which hold them together.

We may thus imagine a working illustration of a ponderable transparent medium of elastic solid type as made up of very small spherical nodules of great density and rigidity dispersed through the æther and imbedded in it. We may even imagine these nodules to be collected into more or less independent groups, each of which will have free periods of relative vibrations of its own nearly independent of other groups, in the manner now well known in connexion with Prof. Ewing's model of a magnetic medium. A wave running across such a medium may excite these groups, and thus illustrate the theory of selective absorption by means of a system in which only the elasticity of the ambient medium is operative, but no other internal forcive.

The explanation of a very weak medium exciting such powerful tractions implies of course strains of enormous intensity, so that its limits of perfect elasticity must be taken enormously wide compared with anything we know in ordinary matter. The magnitude of the strains also requires that the displacement of an atom relative to the æther must be a considerable fraction of its diameter ; and this is sufficiently secured by the large value of ϵ above that which is required to keep down the density of the æther, combined with the great relative density of the atom. It would thus seem to be possible to account for sufficiently large differential tractions between the component atoms of a molecule, especially

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if some of them lie well under the lee of others, to produce brisk internal vibration.

In this way we could imagine the construction of a sort of model illustrative of an elastic solid theory of refraction, including selective absorption and other such phenomena, in the form in which it is presented by von Helmholtz and others. In the simpler case, in which the atoms are not grouped into systems capable of synchronous free internal vibrations, let (ξ, η, ζ) denote the mean displacement of the free æther, and (ξ_1, η_1, ζ_1) that of the atoms. Then the equations of vibration assume the forms

$$\rho \frac{d^2}{dt^2}(\xi, \eta, \zeta) = c\nabla^2(\xi, \eta, \zeta) - \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)p + a(\xi_1 - \xi, \eta_1 - \eta, \zeta_1 - \zeta),$$

$$\rho_1 \frac{d^2}{dt^2}(\xi_1, \eta_1, \zeta_1) = -a(\xi_1 - \xi, \eta_1 - \eta, \zeta_1 - \zeta)^*,$$

in which $p = c'\left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right)$; and the phenomena of crystalline media could be included by assuming a vector-coëfficient instead of the scalar a.

The conclusion, then, is that in this limited range an elastic-solid theory of a very rare æther is not so much at fault as would at first sight appear.

A theory based on difference of rigidity without difference of inertia, after MacCullagh's manner, would have to be realized by ascribing to the atom an atmosphere of intrinsic æthereal strain, instead of endowing it with great inertia ; and this could only be possible in a rotational æther, and would in fact form a mechanical representation of the electric theory. As such it must be expected to give an account of

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* There are introduced by von Helmholtz (*Wiss. Abh.* ii. p. 216) in addition, a forcive proportional to the absolute displacement of the atom, and a frictional one proportional to its absolute velocity. The former is derived from the idea that the heavy central masses of the atoms are unmoved by the æther, and only outlying satellites are affected by its motion. On our present view this restriction might be dispensed with, except in so far as it renders possible an illustrative theory of absorption of an analytically simple character. The consequences of the above equa-, tions are set out by various writers, e. g. Carvallo (*Comptes Rendus*, cxii. p. 522).

the phenomena of electricity as well as those of light*, and in such an account is founded one of its chief claims.

A development of the electric theory has recently been essayed by von Helmholtz†, on the basis of the formal equations of Heaviside and Hertz, in which the free æther is still supposed to be an elastic medium of excessively small density in which the dense atoms are imbedded. If such a view should turn out to be the basis of a consistent body of theory, the considerations given above with respect to the intensities of molecular tractions would have a bearing on it also.

Let us now consider more particularly the explanation that would be offered by the electric theory of light. The difference between a material medium and a vacuum consists in an altered effective dielectric coefficient. This difference is simply and naturally explained by the hypothesis that the material molecules are polar owing to their associated atoms having atomic charges equal in amount but opposite in sign, and that they therefore possess electric moments just as the molecules of a magnet possess magnetic moments. An electric force thus tends to pull the two constituents of a molecule asunder ; and its full intensity is exerted in this manner, not merely its differential intensity over the range of the molecular volume. But a magnetic force has no such tendency even when we take the molecule to be magnetically polarized, because the two poles of a magnetic element cannot be dissociated from each other ; the magnetic moment is thus directly associated with the atom, not with the molecule. In the case of the stationary light-waves the antinodes of the electric force are therefore places where alternating disturbances of a kind suitable to produce decomposition of the molecules are maintained, and may produce strong effects through sympathetic molecular vibration or otherwise ; but at the intermediate antinodes of the magnetic force the individual ultimate atoms may be disturbed by the alternating magnetic force, but there is no tendency to separation of the constituents of the molecule. On the electric theory, therefore, there is abundant justification both for the magnitude of the effect produced, and for

* *Cf.* "A Dynamical Theory . . . ," Phil. Trans. 1884, §§ 122–124.
† Wied. *Ann.* 1894.

its localization as determined by Wiener's experimental in-
vestigation.

The theory, noticed first it seems by Weber, which ascribes
molecular magnetism to the orbital rotation round each other
of ionic charges, and which has very strong recommendations
from the point of view of the dynamics of the æther, may
form a partial exception to this statement. It leaves the
question open as to whether the principal part of the mag-
netic moment is due to orbital motions in the atoms or to the
motions of the constituent atoms in the molecules; though it
suggests strongly the latter alternative. In that case there
will usually be a differential magnetic action of the field as
between these moving atoms; but the magnetic actions on
positive and negative ions will be by no means equal and
opposite, as is true of the electric actions. Thus, for example,
in the limiting case of two equal and opposite ions revolving
round each other, the elements of the equivalent ionic con-
vection-currents will be at each instant parallel, and there will
be no differential magnetic forcive at all; there will also be no
magnetic moment; but the electric differential action will
retain its full force.

It is well understood, and in accordance with this explana-
tion, that the energy of chemical combination of atoms into
molecules is almost entirely that of electrostatic attraction of
their atomic charges. In fact the electric attraction between
them diminishes according to the law of inverse square with
increasing distance, their magnetic attraction according to the
law of the inverse fourth power : if these forces are of the
same order of magnitude in the actual configuration of the
atoms in the molecule, the work done by the former during
their combination must be almost indefinitely greater than
the work done by the latter.

If we contemplate the purely dynamical basis which must
underlie the descriptive explanations of the electric theory of
light, it is difficult to see how there can be any place for a
theory of the æther loaded by the material molecules, which
dynamical views usually associated with Fresnel's theory
demand. There could be no polarity in the inertia of a.
mere load, such as the present considerations require. On
the other hand, the presence of electrically polarized mole-

cules is effectively a diminution of the elasticity of the lumi-
niferous medium ; and I have tried to show elsewhere * that
the principles of MacCullagh's theory of optics are in sub-
stantial agreement with all the general features of our
electrical and optical knowledge.

It is definitely implied in the electromotive, as distinguished
from an electrodynamic, character of the electric theory of
light, that the atomic charges vibrate in unison with the
light-waves, quite unimpeded by any material inertia of their
atoms. This hypothesis is conceivable and natural, indepen-
dently of any particular explanation, on the theory that the
atoms are themselves intrinsic mobile configurations of stress
or motion, or both together, in the ultimate medium.

[It is not without interest to consider how far the conception
mentioned above of an isotropic solid medium of very small
density, with very massive minute nodules imbedded in it but
exerting no direct forcives on each other, will carry us in
forming a representation of optical phenomena. The theory
is of the Young-Sellmeier type, because each nodule has one
or more free periods conditioned by its form and by the
surrounding elasticity.

On eliminating (ξ_1, η_1, ζ_1) from the equations expressed
above, we obtain the vibrational equations of the æther,
supposed thus loaded. Its elastic properties are found to be
conserved intact, but the effective density as regards vibra-
tions of period τ is increased by $a\rho_1 \Big/ \left(a - \dfrac{4\pi^2}{\tau_2}\rho_1 \right)$. When the
coefficient a is of aeolotropic type, by reason either of the
form or the distribution of the nodules, we have effectively
an isotropically elastic medium with aeolotropic inertia ; this
leads to Fresnel's wave-surface, provided the elasticity is
labile in Lord Kelvin's sense. The theory also leads to a
formula for ordinary dispersion, of the usually admitted type
(Ketteler's) for isotropic media ; but, on the other hand, it is
in default by assigning a dispersional origin to double refrac-
tion. If we wish to include the minute effect known as the
dispersion of the optic axes in crystals, it will be necessary to
assume for the elastic stress between æther and matter a

* *Loc. cit.* Phil. Trans. 1894.

somewhat more general form, involving (after von Helmholtz) absolute as well as relative displacement, but always of course remaining linear.

The assumption of elasticity of labile type also allows an escape from the usual difficulties of a solid æther in the matter of reflexion. In that problem the elasticity would naturally be taken continuous across the interface, the volume occupied by the molecules being on this hypothesis extremely small compared with that occupied by the æther.

We may further amend the theory by getting rid of the difficulties associated with lability, at the same time avoiding the difficulty as to how a body can move through a perfect solid medium, if we take the æther to be a rotationally elastic fluid, and retain the material load as before.

But an essential and fundamental difficulty will still remain. It is the extremely small volume-density of the energy involved in radiation which permits a very small inertia, and consequently a small elasticity, to be assigned to the æther, and so prevents it from acting as an appreciable drag or exerting an appreciable force on finite bodies moving through it. But these very properties would incapacitate it for acquiring the very large volume-densities of energy that would have to be associated with it in order to explain electrodynamic phenomena.

Any representation which would make the æther consist of molecules of ordinary matter is open to the objection that the thermal kinetic energy of gases and other material systems must then, in accordance with Maxwell's law of distribution of energy, largely reside in it. But, on the other hand, if we hold to the view of matter which was first rendered precise by Lord Kelvin's theory of vortex atoms, namely, that the æther is the single existing medium and that atoms of matter are intrinsic singularities of motion or strain which belong to it, then there is no inducement to assume for the æther a molecular structure at all, or to make its inertia anything comparable with the inertia of the atoms on whose play the thermal energy of the movements of the matter consists. On such a theory the inertia, and the resulting kinetic energy, of the matter may be hard to explain, but it is certainly something different from the inertia of the underlying

medium in which the atom is merely a form of strain or motion.   On such a theory refraction, and also double refraction, will be caused by the atmosphere of intrinsic strain which represents the electric charge on the atom ; and only dispersion will be assigned to the influence of sympathetic vibrations in the atoms or molecules, thus doing away with any difficulty of the kind mentioned above.

In the theory of gases the ordinary kinetic energy of the molecules represents sensible heat, and as such may be derived for example from the dissipation by friction or otherwise of the mechanical energy of ordinary masses : it is of the nature of kinetic energy of the masses of the atoms.   But the store of energy which keeps up radiation is of electromotive kind, is concerned with displacing electricity, not with moving matter except indirectly ; at least no consistent scheme has yet been forthcoming which includes both.   It is quite conceivable that the disturbances which occur in the ordinary encounters of molecules are of far too gentle a character to excite the very powerful elasticity which on a certain form of the electric theory binds together the continuous medium taking part in optical propagation, any more than a system of solid balls rushing about in an enclosure bounded by a heavy continuous rigid solid can excite sensibly the elastic qualities of that body.   The opinion has been widely supported, both on theoretical and experimental grounds, that a gas will not emit its definite radiations however high the temperature to which it is raised, unless there is chemical decomposition of the molecules going on.   If that be so, the æther does not act as an equalizer of the kinetic energy between the different modes of vibration of the molecules, and the ordinary theory of gases need make no reference to the æther.

If I have understood aright, a similar view has been expressed as at any rate a possible explanation of the difficulty as to the application of Maxwell's distribution theorem in the theory of gases, by Prof. Boltzmann himself.   The law of distribution of energy is perhaps unassailable for the case of molecules like small spheres, with three degrees of freedom, all translational.   By including the rotational modes of freedom, which may be none at all for a monad gas, only two

for a diad, and three for other types, and these possibly not complete, a sufficient number of freedoms is obtained to cover the known range of values of the ratio of the specific heats. The introduction of any vibrational types would make too many ; so on this ground also it is not likely that such types can enter into those among which the thermal energy is divided.—*December* 4.]

DISCUSSION.

A letter was read from Professor LODGE, criticising the manner in which Wiener's results had been summarized by Dr. Larmor, the summary at the commencement of the paper being characterized as misleading. The letter concluded with an expression of admiration for the work done by Dr. Larmor in working out electricity and optics, starting from McCullagh's theory of light.

Dr. STONEY objected to the suggestion that atomic charges could vibrate unimpeded by material inertia. We had not merely to consider the bodily translation of a molecule, but the relative motion of its parts. Radiation absorbed by a gas acted on its atomic charges,—part appearing as translational energy of the molecules, part as energy of their internal motion, and the ether acting as an equaliser of molecular energy.

Prof. MINCHIN did not understand why the polarised light in Wiener's crucial experiment should be incident at 45°. He also found it hard to imagine what vector other than the simple displacement of the ether could be the vector of McCullagh's theory.

Dr. BURTON said he found no difficulty in following Dr. Larmor's statement of Wiener's results ; it appeared to him perfectly clear and satisfactory.

XXVII. *On a Suggestion by* Professor J. J. Thomson *in Connexion with the Luminescence of Glass due to Kathode-Rays. By* John Burke, *B.A., Lecturer in Physics, Mason College, Birmingham.*[*]

Last August, at Oxford, I communicated a paper to the British Association on a strange luminous phenomenon which had been observed by Beccaria more than a hundred years ago. It was there pointed out that although the conclusions arrived at by the Italian physicist, if true, were likely to lead to results of an extremely interesting character, in connexion with Mr. Crookes' important researches on the luminescence of glass, &c., in vacuum-tubes, and although the mysterious nature of the phenomenon was likely to attract much attention, yet the subject was allowed to retain its obscurity. Beccaria ('Artificial Electricity,' § 766) observed that when *vacuum*-bulbs were broken in the dark a light, consisting of a faint glow, was produced in the place where the bulb lay. He attributed an electrical origin to the phenomenon, owing to the manner in which certain substances were supposed to behave in yielding the glow. He mentioned, moreover, that the mere breaking of glass did not give rise to the phenomenon, but that the presence of air was essential to its production, and that when air was allowed to rush suddenly into a vacuum by the bursting of a bladder at the mouth of a receiver, articles such as glass bottles, vessels of sealing-wax, &c., became luminous by, he supposed, the violent dashing of the external air on them ; the luminosity was most conspicuous at the necks of the bottles or at the upper edges of the vessels

As has already been remarked, Beccaria's researches appear to have been forgotten for more than a century, and not until quite recently has attention been drawn to them. This has been done by Professor J. J. Thomson, who, in his ' Recent Researches in Electricity and Magnetism,' p. 119, recalls the fact and indicates its possible close relationship to Mr. Crookes' theory of the luminescence of the glass in Geissler's tubes :

[*] Read November 9, 1894.

that the bombardment of the glass by the particles of gas projected from the kathode is intense enough to cause the glass to become luminous.　Prof. Thomson quotes from Priestley's 'History of Electricity' :—

"Signor Beccaria observed that hollow glass vessels, of a certain thinness, exhausted of air, gave a light when they were broken in the dark.　By a beautiful series of experiments, he found, at length, that the luminous appearance was not occasioned by the breaking of the glass but by the dashing of the external air against the inside when they were broke.　He covered one of those exhausted vessels with a receiver, and letting the air suddenly on the outside of it observed the very same light."

That the light observed in both cases was the same, unless the exhausted vessel, which had been covered by the receiver, was broken by the dashing of the external air against it, is a circumstance which, from considerations that shall presently be adduced, I think we may be justified in questioning.

Through the kindness of Prof. FitzGerald in extending to me the facilities afforded in the Physical Laboratory of Trinity College, Dublin, I have been enabled to experiment upon this subject.　It would be impossible for me to attempt to render in detail the acknowledgment of what has been due to his invaluable advice and suggestions.

It must be mentioned at the outset that the present investigation has by no means been completed, yet, thus far, it appears that some of Beccaria's results have not been wholly of that degree of exactness which we should have hoped for. It must, however, be borne in mind that scientific appliances in his time were less perfect than they are to-day, and such imperfection may account for much inaccuracy.

A number of incandescent lamps of various sizes with broken filaments were procured.　An observer who had been fifteen or twenty minutes in the dark, whose sight had become sufficiently sensitive to the faintest light, broke one of the larger of these bulbs with a hammer, at the same time protecting his face by a plate of thick glass.　A faint *white* glow, which lasted but for a very small fraction of a second, was seen in the place where it was known the bulb had lain.　The experiment was repeated in the presence of several persons,

who testified to the same effect. The intensity of the light seemed to depend upon the size of the bulb—the larger bulbs, about 7 or 8 centim. in diameter, giving a greater intensity than the smaller ones ; whilst with bulbs of not more than 2 centim. diameter no luminous effect was perceived.

A circular plate of glass, about 7 centim. in diameter and 2 millim. thickness, was then placed on the mouth of a receiver, the opening of which was about 5 centim. in diameter, and the contact made air-tight, whilst the air in the receiver was exhausted until a vacuum of about 20 millim. had been obtained.

The plate of glass was then broken with a hammer. A beautiful stream of white light was visible throughout the whole receiver. The light, which lasted for a small fraction of a second, resembled that produced when vacuum-bulbs were broken, though it was more distinct, and consisted not merely of a uniform glow, as in that case, but was marked, in addition, by a number of luminous spots of various sizes : in fact the effect presented the appearance of a faint nebula.

On account of the extreme faintness of the light, it was found impossible to photograph it : at least any attempts in this direction have hitherto proved unsatisfactory, even with the most sensitive plates I could procure.

In some instances the luminous spots were observed to be unusually large ; the fragments of glass were in such cases also found after each experiment to be of an exceptional size, often 7 or 8 square centim. in area.

This fact is especially worthy of notice, as it undoubtedly indicates that the fragments of glass themselves had become luminous, the continuous glow in all probability having been caused by the very minute fragments.

Different kinds of glass were used without giving any perceptible difference in the effects.

Thin brittle plates of cast-iron and steel were tried in the place of glass, but yielded negative results, owing, perhaps, to the fact that they did not break up into innumerable fragments as glass did readily when struck with a hammer, or because the air was not consequently permitted to enter the vacuum very suddenly and with sufficient violence.

Various gases were made to take the place of air. Fig. 1 represents the apparatus employed for this purpose. *a* is a glass receiver ; the strong glass ring *b*, the opening of which was covered by a piece of thin glass *c*, rested on the mouth of the receiver *a*, and served to support a long glass tube *d*, at the upper end of which, being closed, there was an arrangement *e* by which a weight *f* was supported and allowed to fall by turning the handle *h* and thus break the glass *c* and open free communication between the gas in the tube *d* and that in the receiver *a*.

Fig. 1.

The joining between *d* and *a* was made completely air-tight so that no leakage occurred. The tube *d* was then filled with carbon dioxide or oxygen, and the air in the receiver *a* exhausted until a vacuum of about 20 millim. was obtained. The weight *f* was made to fall and break the glass *c*, so that the gas in *d* might enter violently into the vacuum. The same luminous phenomenon was observed as on previous occasions when air was employed, with no perceptible difference as regards colour, intensity, or the general appearance of the glow and luminous spots.

When the gases in both *a* and *d* were exhausted and the glass *c* broken as before, no luminosity whatsoever was observed, thereby showing that the mere breaking of glass did not suffice to produce the phenomenon ; but that the presence of a gas was essential, though any gas was sufficient to produce the effect—that is, that some function performed by a gas in rushing into a vacuum was the cause or a circumstance invariably connected with the phenomenon.

The three following hypotheses seemed possible :—

(1) The violent dashing or bombardment of the molecules of air against the glass might have caused the latter to emit light.

(2) It might have been a sort of miniature meteorite phenomenon, caused by the collisions of the fragments of glass with the interior of the receiver and with each other ; the

intense bombardment of the larger fragments by the minute dust particles giving rise to the luminous spots. ·

(3) It might have been an electrical phenomenon caused by the rubbing of air against glass; somewhat resembling that produced by the friction of mercury against glass in a barometric tube *.

In view of testing hypothesis (1) the following experiments were made :—

A bladder was fastened to the neck of a small receiver, the air in which was then exhausted until a vacuum of about 20 millim. was obtained.

The bladder was then caused to burst, and a stream of *yellowish* light was seen to descend within. This experiment was repeated several times with receivers of various heights; and it was found that in the case of tall receivers the light was confined to the higher portion, near the mouth of the receiver.

The intensity of the light seemed to depend upon the pressure in the vacuum, that is to say, upon the violence with which the air entered the interior of the receiver.

The general appearance of this yellowish light seemed to indicate that the luminosity in *this* case was partly, if not altogether, due to parts of the bladder taking fire, being heated by the friction of the air against them, or by being suddenly torn. Articles of various materials, such as those mentioned by Beccaria, were placed in the interior of the receiver, also a plate of glass supported horizontally so that the air on entering might impinge directly upon it.

A light in some respects, perhaps, resembling that mentioned by Beccaria was observed, but only with short receivers; for in the case of tall ones, under precisely similar circumstances, no light was perceived in the lower region where the articles lay; thus suggesting that in the former case the portions of burning bladder had been stopped, in their fall, by the articles within the receiver. Beccaria took no precautions to determine the part played by the bladder in the phenomenon, or rather, he entirely failed to notice that it ever became luminous.

In order to stop the bits of bladder, gauzes of various

* See Armstrong, Phil. Mag. ser. 3, vol. xviii. (1844).

materials and different thicknesses were tried ; such as should stop the pieces of bladder but permit the air to rush through without much impedance. The glass bottles beneath the gauze, however, were not seen to give rise to any luminosity.

A similar plan was made use of when air was allowed to enter a vacuum by the breaking of glass.

The apparatus employed for this purpose (see fig. 2) consisted of a cylindrical receiver, A ; a ring, *a*, which fitted closely into the cylinder and was supported by iron rods that were adjustable so that the ring could be moved up and down, carried the wire gauze, *b*, which was tightly fastened to it. A plate of glass, *c*, was laid on the mouth of the receiver, and the contact made air-tight ; *d* were articles

Fig. 2.
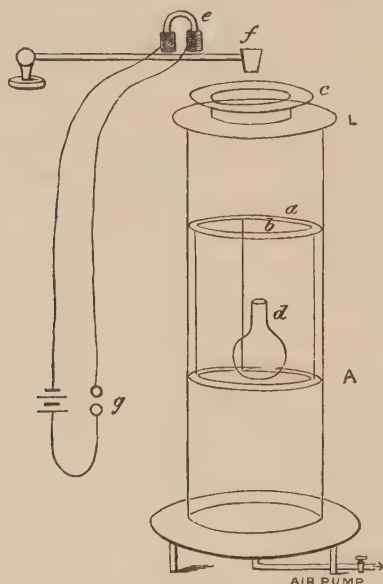
such as glass bottles &c. ; an electromagnet, *e*, supported a hammer, *f*, which fell and smashed the glass plate, *c*, when the circuit was broken at *g*. The part of the cylinder above the gauze was screened from the observer's sight.

The air in the cylinder having been exhausted, no light was seen to penetrate the gauze, neither was there any luminosity seen in the vicinity of *d*, when the external air was

allowed to enter the receiver, in the usual manner, by breaking the glass, c.

A bladder was then used instead of glass plates, but the results were similar.

Different gases were allowed to issue forth from bottles containing these gases in a highly compressed state, on the surface of glass, and especially on its sharp edges, but without producing any visible effect.

These results do not seem to harmonize with hypothesis (1). Let us now proceed to the consideration of hypothesis (2).

Fig. 3 shows the arrangement by which air could be allowed to enter suddenly into a vacuum without the necessity for breaking glass or bursting a bladder. *a* is the glass receiver; *b* a ring, between which and the receiver is a sheet of fine tissue-paper, *c*, on which a number of fragments of glass were placed; a plate of strong glass situated on top of the ring *b* admitted of being drawn aside, and thus allowed the air to enter the vacuum, whilst the rest of the apparatus was held fast.

Fig. 3.

A luminosity similar to that seen upon previous occasions was observed, though less distinct in some respects, perhaps, than before.

Under precisely similar circumstances, in the absence of any fragments of glass on the tissue-paper, no light was produced, whether or not there were any articles that the air could impinge upon within the receiver.

A single piece of glass cut into a definite shape was then made to take the place of the great number of fragments, some cotton-wool being placed on the plate of the air-pump in order as far as possible to prevent the piece of glass from breaking.

The following facts were noticed :—(α) Only a single spark seen. Not as on a previous occasion when a number of fragments of glass were allowed to enter the vacuum,
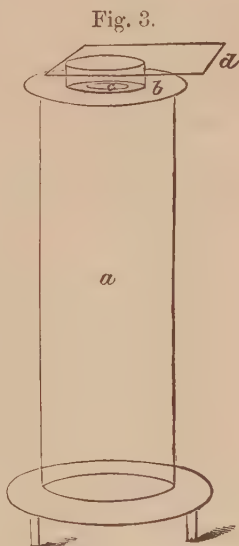
(β) the spark always appeared on the side of the receiver in the direction in which the plate d had been drawn aside * (see fig. 3). (γ) The piece of glass was found almost vertically under the place where the spark had taken place.

It seemed probable that the spark was caused by the collision of the piece of glass with the interior of the receiver. Two strong pieces of glass were then struck violently together, and a light similar in every respect to that previously observed was produced.

These experiments seem to justify hypothesis (2) : the dust particles causing the glow by their impacts against the interior of the receiver and against each other ; whilst the brightness of the luminous spots may be attributed to sparks produced by the larger fragments of glass, and to the bombardment of the dust particles against the surfaces of these larger fragments.

A large quantity of fragments of cast-iron, steel, ebonite, sealing-wax, bone, &c., were tried as well as glass, but the attempt to obtain a luminous effect by the above method proved fruitless.

If metallic dust should produce a luminous effect when made to bombard the surface of glass, and if different effects should be produced by lead glass and German glass, the theory that *solid* particles emitted from the kathode would give rise to luminescence of the glass would be justified. The above experiments, however, do not seem to throw much light on Mr. Crookes' theory of luminescence due to the emitted *particles of gas.*

In the earlier stage of the investigation hypothesis (3) appeared plausible enough. Any experiments, however, that were made in connexion with it did not lead to results of an interesting character.

Whatever may be the real value of these experiments, it cannot be doubted that the inquiry into the cause of such a phenomenon as that noticed by Beccaria must prove of the greatest interest : firstly, on account of the obscurity enveloping the subject, and, secondly, on account of the interest attached to the artificial production (though on a small scale) of meteoric phenomena in the laboratory.

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* It is probable *à priori* that this should take place.

XXVIII. *Students' Simple Apparatus for Determining the Mechanical Equivalent of Heat. By* Prof. W. E. AYRTON *and* H. C. HAYCRAFT*.

I. *The Object to be Attained.*

SOME time ago the authors considered the possibility of constructing an apparatus for the determination of the mechanical equivalent of heat which could be placed in the hands of junior students, and which would enable a sufficiently accurate result to be obtained without the introduction of troublesome corrections. For such a purpose the electrical method was naturally adopted ; for now that the commercial values of the electrical units are known with considerable accuracy in the C.G.S. system, it is possible to measure energy in foot-pounds by means of a good commercial ammeter voltmeter and watch with greater ease and certainty than by any mechanical dynamometer.

Further, it has become easy to obtain as much electric power as is wanted for the experiment at a small cost, for the price of half a horse-power for ten minutes is only one-third of a penny, at 6*d.* per Board of Trade Unit. Hence there is not the practical objection to the electrical method that was so formidable when currents of 20 or 30 amperes could only be obtained by setting up a large battery of Grove or Bunsen cells. Indeed every properly organized physical laboratory is now provided with accumulators, from which a quarter or half a horse-power may be readily obtained for use in such experiments as those to be described ; or, when accumulators are not available, power may be taken from the mains of one of the numerous electric-supply companies.

II. *Design of the Apparatus.*

The authors therefore set themselves to design an apparatus which, when used with a good commercial ammeter voltmeter, thermometer, and watch, would give the value of the mechanical equivalent of heat correct to one per cent. without any

* Read November 23, 1894.

x 2

corrections having to be made even for the heat lost by radia-
tion, convection, and conduction, and without any special
manipulative skill being required on the part of the observer.

Broadly, the experiment consists in passing a known cur-
rent through a resistance immersed in a known mass of water,
and measuring the rise of temperature in a given time, and
the average value of the P.D. between the terminals of the
resistance.

It is evident that if the loss of heat during the experiment
is to be small enough to be neglected in comparison with the
quantity of heat generated, either the rise of temperature
must be small, or must take place in a very short time, or
the ratio of the cooling surface to the mass of the water
heated must be small. As it is impracticable to reduce either
the rise of temperature or the time to very small limits, and
also to measure them with ease and accuracy, it is clear that
the result is best obtained by using a large quantity of water,
for the ratio of the surface area to the mass can then be made
small enough to bring the error due to cooling within the
required limits. But a large quantity of water necessarily
involves the use of a large amount of electrical energy; and
we thus arrive at the result that the accuracy attainable
depends upon the amount of *power* at our disposal, and will
be greater the greater the electric power that can be supplied.

In designing such an apparatus, then, the first thing to
determine is the amount of electric power that can be used,
and the details should then be arranged so as to get the least
error in the result. In those cases in which an increased
accuracy in one measurement involves a diminished accuracy
in another, it is best to make the errors due to the two causes
equal. For example, suppose that the time during which the
electric energy is supplied is such that we can only measure
it to one per cent., while the error due to cooling during the
experiment is only $\frac{1}{2}$ per cent. We can clearly increase the
accuracy of the result if we increase the time until the pro-
bable error in reading it is equal to the error due to cooling
during that time, say $\frac{3}{4}$ per cent. If we were still further to
increase the time, the error due to cooling would increase ·
and exceed $\frac{3}{4}$ per cent., and our result would therefore be less
accurate. This equality of course does not apply to errors

that are not interdependent, such as errors in reading volts and amperes : each of these errors should independently be be made as small as possible.

The measurements to be made are as follows :—

(a) The value of the constant current passed through the resistance.

(b) The average value of the P.D. between its terminals.

(c) The mass of water heated, to which must be added the water-equivalent of the containing vessel, resistance-coil, and stirrer.

(d) The rise of temperature of the water.

(e) The time during which the current is passed.

With the excellent electrical measuring-instruments now obtainable it is possible to measure either current or pressure with an accuracy much greater than one per cent.; indeed the Board of Trade undertake to measure them within one tenth part of one per cent.* We can also measure the mass of water with considerable accuracy : any error, even a large one, made in determining the water-equivalent of the other bodies raised in temperature becomes of small consequence when the water-equivalent is added to the much larger and accurately measured mass. The measurements (a), (b), and (c), there-fore, give us little trouble and do not affect the design of the apparatus.

The case of (d) and (e) is different. In order to measure a change of temperature by means of a thermometer with the accuracy required, either the change must be fairly large or the thermometer must be very sensitive ; but as it is of little use to employ an exceedingly sensitive thermometer to mea-sure the temperature of a liquid which is being locally and rapidly heated, even if the stirring is very efficient, it is necessary to use a fairly large rise of temperature. Similarly, if the circuit is to be closed and broken by hand, and the interval of time measured by an ordinary stop-watch such as would be found in a junior laboratory, there is a certain minimum time required to give the required accuracy of measurement. Also we must keep the ratio of heat lost to heat generated during the experiment equal to the probable

* See Schedule to Final Report of Electrical Standards Committee, 1894.

error in the time or temperature-measurement; for as these three quantities are interdependent, the best condition is to make the percentage accuracy of the temperature measurement, the percentage accuracy of the time measurement, and the percentage heat lost of heat generated, equal.

Let W be the maximum number of watts at our disposal ;

M the mass of water, including water-equivalents ;

S the area of surface of the containing vessel ;

ϵ the average emissivity of the cooling surface, or the ratio of heat lost in calories per second to the surface area, for 1° C. excess temperature ;

Θ the minimum change of temperature that can be measured to 1 per cent. under the prescribed conditions ;

T the minimum time, in seconds, between closing and opening the switch that can be measured by the stop-watch to 1 per cent. ;

θ the rise of temperature of the water ;

t the time during which the switch is closed ;

and let $\frac{1}{x}$ per cent. be the maximum accuracy of result obtainable under these conditions.

Since $\theta = x\Theta,$

and $t = xT,$

and the ratio of heat lost to heat received is $\frac{1}{100\,x}$,

we have

$$\frac{S\,.\,x\frac{\Theta}{2}\,.\,\epsilon\,.\,xT}{M\,.\,x\Theta} = \frac{1}{100\,x},$$

or

$$x^2 = \frac{2M}{100\,S\,.\,\epsilon\,.\,T}. \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

We also know, say from preliminary experiments, that the heat received in calories is about 0·24 time the energy in watt-seconds.

$$Mx\Theta = 0{\cdot}24\,W\,xT,$$

or

$$M\Theta = 0{\cdot}24\,WT. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In equations (1) and (2) M and x are the only unknown

quantities, for S is a function of M depending on the shape of the vessel containing the water. Thus for a cylindrical vessel of height equal to its diameter,

$$S = 5\cdot53\,M^{\frac{2}{3}};$$

while for a spherical vessel,

$$S = 4\cdot84\,M^{\frac{2}{3}}.$$

We can therefore find both M and x from the equations, and since

$$\theta = x\Theta,$$

and $$t = xT,$$

θ and t are also determined.

It may be observed here that as S varies as $M^{\frac{2}{3}}$, and x^2 varies as $\dfrac{M}{S}$, x is proportional to the 6th root of M or to the 6th root of W. Hence if the number of watts available is doubled the accuracy of the experiments is by no means doubled, but is only increased by about $\frac{1}{8}$th.

In the particular case for which the apparatus was designed the number of watts available was about 300, the maximum current being 30 amperes. This determined the resistance of the coil or strip as $\frac{1}{3}$ of an ohm.

The average emissivity for small excess temperatures of a glass vessel standing on a felt base and containing water was obtained by taking cooling curves, the mean value being 0·000232 calorie per square centimetre of area per 1° C. excess temperature. The water was kept at a uniform temperature by means of a light wooden stirrer during these experiments.

Θ, the minimum rise of temperature that can be measured to one per cent., was taken as 2·5 degrees, as it was not considered advisable to rely on the temperature measurements to more than $\frac{1}{40}$ of a degree. The thermometer used is read without the aid of a telescope, and is graduated in 20ths of a degree.

T, the minimum time that can be read to one per cent., was taken as a minute and a half, as it was thought that an error of nearly a second might be made in the measurement of the time between closing and opening the switch, and stop-watches are often a little doubtful as to their zero.

From equation (2) we have at once

$$M = \frac{0{\cdot}24 \times 300 \times 90}{2{\cdot}5}$$

$$= 2592 \text{ cubic centimetres.}$$

Substituting in equation (1), we have

$$x^2 = \frac{2}{100} \times \frac{\sqrt[3]{2592}}{5{\cdot}53} \times \frac{1}{{\cdot}00023} \times \frac{1}{90},$$

$$x = 1{\cdot}55 ;$$

or the accuracy of measurement is $0{\cdot}64$ of one per cent.   The rise of temperature adopted should therefore be $3{\cdot}87$ degrees, the time being 2 minutes 20 seconds.   The numbers actually adopted in the experiments were 2000 cubic centim. of water and a time of 2 minutes ; the smaller quantity of water being adopted because the resistance of the immersed strip when made proved to be rather less than $\frac{1}{3}$ ohm, the watts taken at 30 amperes being 260 instead of 300.

To ensure success in an experiment of this kind it is necessary that the water be as uniformly heated as possible, and that the stirring be very efficient; and after some consideration the authors decided that the best of all plans would be to use a movable conductor of considerable surface, and so shaped that it might itself be used to stir the liquid.   By this means an exceedingly uniform rise of temperature may be produced, every particle of water receiving heat direct from the strip at practically the same rate.   The following is a description of the apparatus as constructed by the authors.

A strip of manganin (chosen on account of its low temperature-coefficient), about $\frac{1}{4}$ inch wide, $0{\cdot}03$ inch thick, and about 5 feet long, was bent into a series of zigzags (fig. 1) so as to form a kind of circular gridiron, M M, in which the successive portions of strip lie all in one plane, the whole being held rigid by a strip of vulcanized fibre F crossing the gridiron and to which each portion of strip was screwed.   Another precisely similar gridiron was placed 3 inches below the first, and they were held together by three thin ebonite pillars E screwed to the strips of vulcanized fibre, the whole forming the top and bottom of a sort of cylindrical box, 5 inches

diameter and 3 inches high. The two grids were joined in series, and the ends of the strip of manganin were soldered to two copper wires C C, about 0·128 inch thick and 6 inches

Fig. 1.
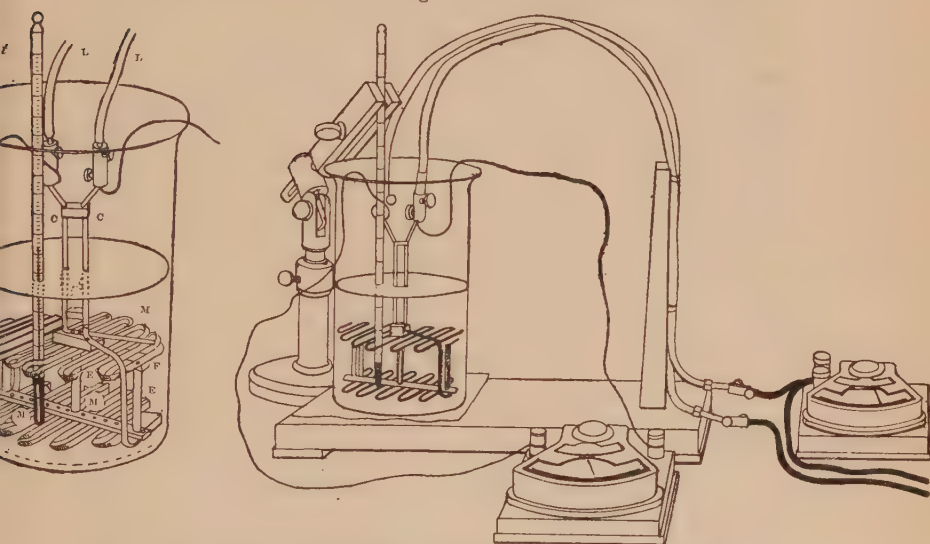
long, which were insulated from one another by vulcanized fibre separators, and constituted a kind of handle by means of which the open box could be moved up and down in the containing vessel.

The whole surface exposed to the water received a thin coat of varnish to prevent any electrolysis due to the difference of potential between the different parts of the strip. At two points in the same vertical line the zigzags forming the two grids are bent so as to leave a space for the passage of a thermometer t, the bulb of which is midway between the grids when the lower one is resting on the bottom of the vessel. The whole heating surface exposed to the water is about 60 square inches, or 400 square centimtres. The vessel used to hold the liquid is a thin glass beaker of just sufficient diameter to take the framework of manganin strip.

Electric connexion with the two stiff wires which form the handle is made by means of two well-insulated very flexible leads L L, each composed of a strand of about 210

copper wires ·011 inch diameter. The size of the stout
copper wires and of the flexible lead was chosen with the
intention that there should neither be heat received nor heat
lost on account of these connexions. The sectional area of
the insulated flexible lead is rather greater than that of the
stout copper wires, and the ultimate rise of temperature of
both with a current of 30 amperes is about 7° C.; the rise of
temperature during the short time that the current is passed
being thus about equal to that of the water, and automatically
preventing gain or loss of heat. The number of watts taken
by the copper wires and flexible leads at 30 amperes is
about 1·9.

It may here be pointed out that it is far better, from the
point of view of getting a good mechanical design, to use a
large current at a low pressure than to take the same power
from a small current at a considerable pressure. Thus it is
easy to design a strong stirrer of the shape described above
which will have a resistance of $\frac{1}{3}$ of an ohm, carrying 30
amperes at 10 volts pressure; but it would be by no means
so easy to make an equally efficient and substantial stirrer of
33·3 ohms resistance to take 3 amperes only at 100 volts.

It was thought worth while to ascertain to what extent
conduction through these leads influenced the rate of cooling
of the vessel when no current was passing, and cooling curves
were therefore taken with the manganin framework in the
vessel and attached to the leads. The average rate of cooling
was found to be 0·000242 calorie per square centimetre area
of surface per 1° C. excess temperature; which is only about
4 per cent. greater than in the previously mentioned experi-
ments, in which a light wooden stirrer only was used, the
value then obtained being 0·000232.

III. *Use of the Apparatus.*

In the following table are given the results of several
successive experiments made with the apparatus described
above by Messrs. Solomon and Grogan, students at the Central
Technical College, under our supervision. The amperes and
volts are expressed in the international units adopted at the
Chicago Congress. The ammeter and voltmeter used were

the well-known Weston-d'Arsonval type instruments. The current was known within about $\frac{1}{3}$ of one per cent., and the average potential-difference within about $\frac{1}{5}$ of one per cent., the instruments having recently been tested against our standards at the Central Technical College. 2000 cub. centim. of water were used in all the experiments, and were measured with a graduated glass jar, fresh water being used in each experiment. The weight of the glass vessel up to the water-level was determined as 184 grammes, and the specific heat being about 0·2, the water-equivalent is 37 grammes. The weight of the manganin strip is 114 grammes, and the specific heat being about 0·09, the water-equivalent is 10; the equivalent mass of water used being therefore 2047 grammes.

No. of expt.	Time in seconds.	Temperature.			Amperes.	Mean volts.	Calories per watt-second.
		Initial.	Final.	Rise.			
1...	120	18·40	22·02	3·62	30	8·634	0·2383
2...	180	13·25	18·70	5·45	30	8·634	0·2390
3...	180	13·60	19·00	5·40	30	8·648	0·2367
4...	120	12·97	16·53	3·56	30	8·656	0·2375
5...	120	12·64	16·26	3·62	30	8·698	0·2365
6...	120	12·89	16·49	3·60	30	8·662	0·2364
7...	120	12·11	15·72	3·61	30	8·666	0·2368
8...	120	12·10	15·74	3·64	30	8·642	0·2395
9...	120	13·13	16·75	3·62	30	8·692	0·2367

Mean **0·2375**

Average deviation from the mean = ·001 = 0·42 per cent.

With this apparatus, then, we get in about 10 minutes, including the experiment and subsequent calculation, a result for the heat-equivalent of the watt-second ; and these results have an average deviation from the mean, if several experiments are made, of less than $\frac{1}{2}$ of one per cent. This result, as we have seen above, should not differ from the true value by as much as one per cent. It may here be pointed out that as the water taken from the ordinary water-supply is sure to be at least two or three degrees below the atmospheric temperature at the time, it is exceedingly easy to arrange that its mean temperature during the experiment shall be exactly equal to the temperature of the air, for no artificial method of cooling is required, as would be the case if a

greater range of temperature were used. This, however, was not done except in one or two of the above experiments, and is not necessary to ensure the accuracy that was required.

The mean temperature of the water in the above experiments was 15 degrees Centigrade; and our final result is that at that temperature 0·2375 gramme of water are raised 1° C. in temperature by the energy of one watt-second.

Now we know with considerable accuracy the equivalent in ergs of one watt-second, which certainly does not differ from 10^7 by more than one tenth of one per cent.; that is to say, the international volt and ampere do not differ from 10^8 and 10^{-1} C.G.S. units of electromotive force and current by more than that amount (see section on the recent history of the electrical units). Hence the result we have obtained, the error in which is less than one per cent., gives us at once a value for the mechanical equivalent of heat with the same accuracy.

Since we find the equivalent of the watt-second in gramme-degrees at 15° C. to be 0·2375, we have at once, taking 10^7 ergs equal to one watt-second, the mechanical equivalent of heat in ergs per gramme-degree at 15° C. equals $4·211 \times 10^7$.

Reducing this to foot-lbs. at Greenwich per lb.-degree C. at 15° C., it becomes **1408**, or in foot-lbs. per lb.-degree F. at 59° F. it is **782**.

A series of four experiments made subsequently by some students at the Central Technical College, in which the cooling error was eliminated by making the mean temperature of the water equal to that of the air, gave as their mean 0·2384 calorie per watt-second, or $4·195 \times 10^7$ ergs per gramme-degree at about 15° C. Reducing this value to foot-lbs. at Greenwich per lb.-degrees C. and F. respectively, we get 1403 and 779 foot-lbs.

IV. *Previous Determinations of the Mechanical Equivalent of Heat.*

It is of interest to compare these figures with some of the more recent results obtained for the mechanical equivalent.

Rowland's value for the mechanical equivalent in ergs per gramme-degree at 15° C. is $4·189 \times 10^7$, which reduced to

foot-lbs. at Greenwich per lb.-degree C. is 1401, and in foot-lbs. per lb.-degree F. at 59° F. is 778·3.   Rowland used the method of direct friction of water, and was the first to discover that the specific heat of water was a minimum at 30° C. and varied one per cent. between 5° and 30° C. (Proc. American Acad. 1879–80).

Dieterici (1889) gives as the result of a determination by the electrical method in which Bunsen's ice-calorimeter was used, the number $4·244 \times 10^7$.   Correcting from the " legal " ohm employed by him to the international unit, we get the numbers $4·232 \times 10^7$, 1415, and 786 (*Annalen der Physik*, vol. xxxiii. p. 417).

Miculescu's careful determination by the direct method in 1892 gives $4·186 \times 10^7$ on the mercury scale of hard glass, or 4·187 on the hydrogen scale, at about 12° C.   The latter number expressed in foot-lbs. at Greenwich per lb. degree C. is 1400, and in foot-lbs. per lb. degree F. is 778.   A platinum non-thermal junction was used in these experiments, and was calibrated against a Gonnelot standard mercury thermometer.

Griffiths, using the electrical method, gives as his final result (Proceedings Royal Society, vol. iv. p. 26), $4·198 \times 10^7$, the corresponding numbers in the other units being 1404 and 780.

The recent determination of Schuster and Gannon, also by the electrical method, gives $4·180 \times 10^7$ on the mercury scale of hard French glass, or 4·192 on the hydrogen scale, at a temperature of 19°·1 C.   The latter value expressed in foot-lbs. at Greenwich gives 1402 and 779.

Joule's values obtained by friction of water in 1878 gave the numbers $4·159 \times 10^7$, 1390·5, and 772·5 respectively (mercury scale); while his values obtained by the electrical method in 1867, when corrected for the error in the B.A. ohm, give $4·155 \times 10^7$, 1389, and 772, the average temperature of the water being 19° C.   The following table gives the results obtained by various methods since 1867, the numbers having been recalculated from those given in a table at the end of M. Miculescu's paper (*Ann. de Chimie*, vol. xxvii. p. 202) :—

Date.	Observer.	Method.	Ergs per gramme-degree C.	Foot-lbs. per lb.-degree F.
1867.	Joule.	Electrical.	4.155×10^7	772
1870.	Violle.	Heating of a disk between the poles of a magnet.	4.269×10^7	793
1875.	Puluj.	Friction of metals.	4.179×10^7	776
1878.	Joule.	Friction of water.	4.159×10^7	772
1878.	Weber.	Electrical.	4.145×10^7	770
1879.	Rowland.	Friction of water 15°.	4.189×10^7	778
1888.	Perot.	By the relation $L = \tau(v_2 - v_1)\dfrac{dp}{dt}$.	4.167×10^7	774
1889.	Dieterici.	Electrical.	4.232×10^7	786
1891.	D'Arsonval	Heating of a cylinder in a magnetic field.	4.161×10^7	773
1892.	Miculescu.	Friction of water.	4.186×10^7	778
1893.	Griffiths.	Electrical.	4.198×10^7	780
1894.	Schuster and Gannon.	Electrical.	4.192×10^7	779

The results obtained since 1879 by stirring water and by electrical methods may be tabulated thus :—

$$
\begin{aligned}
&\text{Rowland (friction)} \quad && 4.189 \times 10^7 \text{ or } 778, \\
&\text{Dieterici (electrical)} \quad && 4.232 \times 10^7 \text{ or } 786, \\
&\text{Miculescu (friction)} \quad && 4.186 \times 10^7 \text{ or } 778, \\
&\text{Griffiths (electrical)} \quad && 4.198 \times 10^7 \text{ or } 780, \\
&\left.\begin{array}{l}\text{Schuster}\\\text{and}\\\text{Gannon.}\end{array}\right\} \text{(electrical)} && 4.192 \times 10^7 \text{ or } 779.
\end{aligned}
$$

The result obtained by our students is greater than the mean of these results by just one half per cent. It is probably not generally known that such accuracy can be obtained by the use of such simple apparatus as that here described, and the fact is we think sufficient justification for bringing the result under the notice of the Society.

V. *Recent History of the Practical Electrical Units.*

A short account of the recent history of the practical electrical units is added for the benefit of those not familiar with the work of the Board of Trade Standards Committee.

The first report of the Electrical Standards Committee, appointed by the Board of Trade at the end of 1890, was presented in July 1891, and contained the following recommendations among others :—

" 4. That the resistance offered to an unvarying electric current by a column of mercury of a constant cross sectional area of one square millimetre and of a length of 106·3 centimetres, at the temperature of melting ice, may be adopted as one ohm."

" 10. That an unvarying current, which when passed through a solution of nitrate of silver in water, in accordance with the specification attached to this Report, deposits silver at the rate of 0·001118 of a gramme per second, may be taken as a current of one ampere."

At the British Association meeting held at Edinburgh in August 1892 the B.A. Committee on Electrical Standards, in consultation with Dr. von Helmholtz, M. Guilleaume, Professor Carhart, and others, agreed to the following resolutions among others :—

" 2. That 14·4521 grammes of mercury in the form of a column of uniform cross section 106·3 centimetres in length at 0° C. be the specified column of mercury adopted as the practical unit of resistance."

" That the number ·001118 should be adopted as the number of grammes of silver deposited per second from a neutral solution of nitrate of silver by a current of 1 ampere, and the value 1·434 as the electromotive force in volts of a Clark cell at 15° C."

These decisions agreed with those which had already been laid before the German Government by Dr. Helmholtz and the Curatorium of the Reichsanstalt. It was considered preferable to define the mercury column in terms of length and mass, as the sectional area cannot be directly measured with such accuracy; but the length 106·3 centimetres was adopted so that the sectional area might be, as nearly as could be known, one square millimetre, as this area has been specified in all previous definitions.

In November 1892 a supplementary report was presented to the Board of Trade by its Standards Committee, and included the following resolutions :—

" 4. That the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14·4521 grammes in mass of a constant cross sectional area, and of a length of 106·3 centimetres, may be adopted as one ohm."

"10. That an unvarying current which, when passed through a solution of nitrate of silver in water, in accordance with the specification attached to this Report, deposits silver at the rate of 0·001118 of a gramme per second, may be taken as a current of one ampere."

" 14. That the electrical pressure at a temperature of 15° Centigrade between the poles or electrodes of the voltaic cell known as Clark's cell, prepared in accordance with the specification attached to this Report, may be taken as not differing from a pressure of 1·434 volts by more than one part in one thousand."

At the International Electrical Congress held in August 1893 at Chicago these values were adopted by the delegates as those to be recommended to their several governments as legal units.

The Board of Trade Committee, after having had laid before them the recommendation of the Electrical Congress, which was substantially that contained in their supplementary report of November 1892, presented their final report advising a legalization of the standards as defined, by an Order of Her Majesty in Council ; and on August 23rd, 1894, this was effected, and the practical standards we may hope permanently settled.

Of the three units—ohm, ampere, and volt—the two former are defined on the C. G.S. system, and the latter as a secondary unit from the relation of the first two. The numbers given in the above-quoted resolutions as to the ohm and the ampere are the values which appear to represent most accurately the true volt and the true ampere. In the Report of the B.A. Standards Committee for 1892 is given a table showing the results of nine determinations of the ohm expressed in centimetres of mercury, made between 1882 and 1892; the mean of these results being 106·31, and the average deviation from this mean being ·015, or one part in 7000. We may therefore safely say that the ohm, as now defined, does not differ from 10^9 C.G.S. units of resistance by more than one part in two or three thousand.

As regards the ampere, we have the very careful determination of Lord Rayleigh and Mrs. Sidgwick giving ·0011179 gramme of silver per coulomb, that of Kohlrausch ·0011183,

Gray ·001118, and Potier and Pellat ·0011192. It is at least probable that the value ·001118 adopted by the Board of Trade Committee does not differ from that corresponding to the absolute C.G.S. unit divided by ten by more than one part in a thousand.

Hence it is probable that the number of ergs equivalent to one watt-second does not differ from 10^7 by more than one part in a thousand; and that being the case it would seem that, considering the great accuracy with which electrical measurements can be made, the electrical method of determining the mechanical equivalent of heat should be the most accurate.

DISCUSSION.

Mr. GRIFFITHS thought it unadvisable to provide junior students with apparatus from which every source of error had been eliminated ; they were thus led to underrate the difficulty of an experiment and the care required to obtain reasonable accuracy.

Prof. CAREY FOSTER agreed generally with Mr. Griffiths, and deprecated the use of direct-reading am-meters and volt-meters in experiments of this kind.   He thought it preferable that a student should learn to reduce instrumental readings to absolute measure for himself.

Prof. S. P. THOMPSON dissented from the opinions expressed by the two previous speakers, and thought it was an advantage to students to have the use of direct-reading instruments.

Dr. SUMPNER described a simple method which he had employed for measuring the mechanical equivalent of heat, and which depended on the heating of a stream of water as it flowed through a pipe containing the current conductor.

Prof. RÜCKER was inclined to take an intermediate view of the questions that had been raised.   He thought that students should take for granted as little as possible concerning their instruments, but to verify every point, even if practicable, would occupy much time that might be otherwise more profitably employed.

Prof. AYRTON replied, and explained that the calibration of am-meters and volt-meters would be part of the work of a student at another part of his course.

XXIX. *Mirrors of Magnetism.*

By SILVANUS P. THOMPSON, *F.R.S., and* MILES WALKER[*].

OUR knowledge of Electric Images is based almost entirely
upon mathematical deduction. It would be difficult to
satisfactorily demonstrate their properties by experiment.
We are content to take the elegant geometrical proofs that
Lord Kelvin has given us, and corroborate these physically at
points where the conclusions are within reach of experiment.
The subject, therefore, is one of purely theoretical interest,
but so beautifully does it elucidate difficult problems in Elec-
trostatics that it remains the continual delight of text-book
writers.

Magnetic Images, on the other hand (apart from one or

Fig. 1.
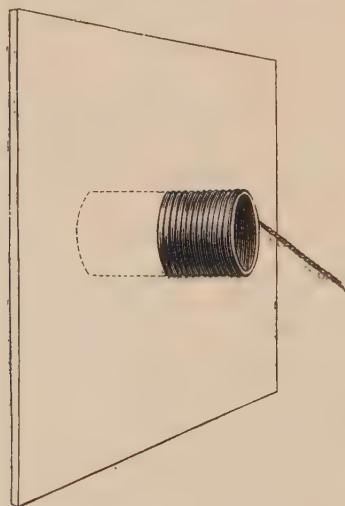
two investigations by the same great master), are entirely
neglected, and yet these are much more likely to be practi-
cally utilized in the near future than their electrical analogues.
It may be in questions of dynamo and instrument shielding,
or it may even be in the solution of that commercial problem,
the testing of the magnetic properties of iron *en masse.*

[*] Read November 28, 1894.

The existence of magnetic images is suggested by considerations altogether apart from the mathematical.

If we take a solenoid of wire through which a current is flowing and place its end against a large plate of iron, we eliminate the effect of that end upon the distribution of the field about the coil, and so obtain a field on one side of the plate of a shape the same as if the coil were continued to double its length. If we place the coil with its end against a mirror (fig. 1), we see it reflected in the mirror so that it appears to be double its real length. Thus the iron plate gives us magnetically an image of the coil analogous to the optical image produced by the mirror, geometrically identical indeed with the optical image, virtual and erect.

Fig. 2.
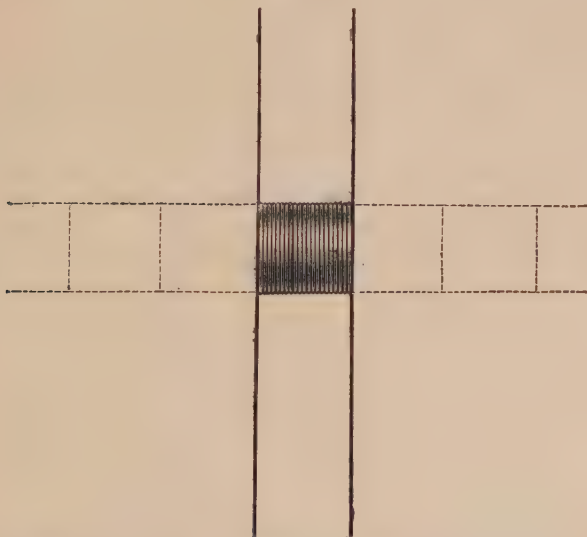
If a large iron plate is placed at each end of the coil the field will be the same as if the coil were infinitely long. We know that two mirrors, facing each other, one at each end of the coil, would give it the appearance of stretching away to infinity in both directions (fig. 2).

Of course these optical effects could only be perfectly obtained if the mirror were a perfect reflector; and to include all possible points of view it would have to be infinitely large. So the magnetic effects could only be perfectly

obtained if the plates were perfectly permeable and infinitely large. Observe that we are only considering the effect at a point *in front* of the mirror and in front of the iron plate. *Behind*, that is to say on the side opposite the coil, there is no effect in either case.

It was thought interesting to ascertain how far effects of this kind could be obtained in practice, using an ordinary iron plate somewhat less than that required for the perfect case. For this purpose two similar solenoids were wound, 5 centim. long and 4 centim. mean diameter, the winding at the ends being exposed so that it could be put up close to the iron. A search-coil of 100 turns, about $1\frac{1}{2}$ centim. in diameter, was mounted so that it could be fixed at different positions relatively to one of the solenoids, which was then

Position of Search-coil.	Throw when coil on coil.		Throw when coil on iron.	
	Make.	Break.	Make.	Break.
	236	236	236	235
	235	235	234	234
	230	230	230	229
middle	227	227	228	227
	204	202	204	202
end	129	126	129	127
	69	67	69	68
	41	41	41	41
	24	24	24	23

placed upon the iron plate* and upon the other solenoid
alternately.   A current of constant value was turned on and
off in each position of the solenoid, and the throw on a ballistic
galvanometer in the circuit of the search-coil observed.   The
preceding table, giving the throw of the needle for different
positions of the search-coil, shows how perfectly the iron
plate mirrored the magnetism of the solenoid.   The dis-
crepancy in the figures arises more from errors of observation
than anything else.

The first position in the above table is the most important,
as the coil is placed in a part of the field that experiences the
greatest change by the presence or absence of the iron plate,
the difference being that between 236 and 129.   If the search-
coil could have been placed so that its centre was at the very
end of the solenoid, the throw ought to be just one half of
what it would be with the coil continued to double its length.
This was found to be so when the solenoid was placed on the
other solenoid, but could not be tried with the iron as part of
the search-coil protruded.

The experiment was then made of lifting the solenoid step
by step a little distance from the iron plate, when it was
found that the presence of the iron plate had the effect of
producing an image of the coil receding an equal amount
behind the plate.

Fig. 3.
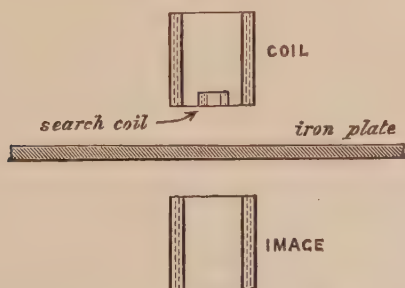
A larger search-coil was placed in different positions out-
side the solenoid ; and in all positions on which any readable
throw of the needle could be obtained, it was found that the

* The iron plate was ordinary boiler-plate $\frac{1}{2}$ inch thick, size 3 feet by
2 feet.

iron plate gave an effect almost exactly the same as an equal
solenoid in the position of its image.

In order to try the effect of tilting the solenoid, it was
mounted on a strip of wood hinged to another, the angle

Fig. 4.
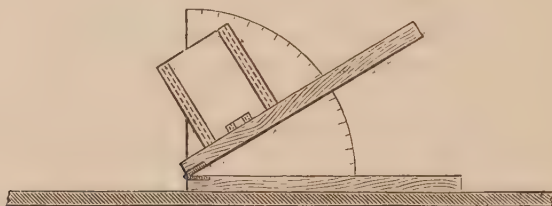
between them being fixed by a clamp (fig. 4.)   This was then
placed alternately on the iron plate and against a similar
hinged strip upon which the other solenoid was placed, the
angle of inclination being the same for both solenoids (fig. 5).

Fig. 5.
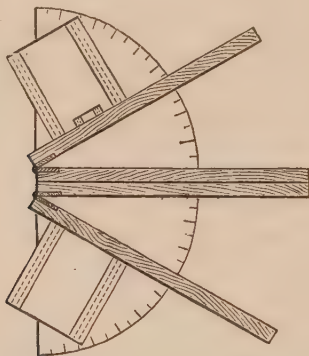
The angle was altered step by step from 0 to 90 degrees, and
in all cases the iron plate produced the same effect as its
image within limits not greater than those shown in the
previous table.   It was, however, only when the solenoid
was placed at the end of the hinged strip nearest the iron
plate that readings of any value could be obtained, as small
dislocations in the position of the coil laterally were apt to
produce changes of reading comparable with those due to
considerable changes in angle.   At distances greater than

5 centim. the iron had very little effect. The search-coil
during these experiments was moved about to different
positions both inside and outside the solenoid, preference
being given to the most sensitive positions.

Then the effect of differently-shaped solenoids was tried.
A long narrow one and a wide flat one were used with similar
effects, so far as they could be measured.

There is a difference between the magnetic reflexion of an
iron plate and the optical reflexion of a mirror in the following

Fig. 6.
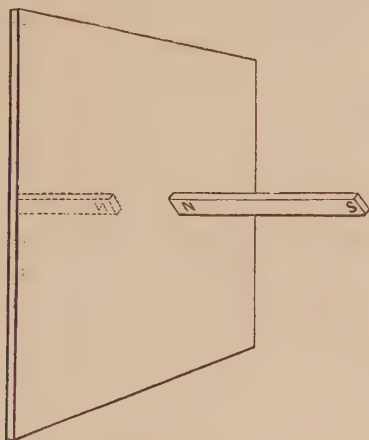
respect : an optical image has those of its lines which are at
right angles to the mirror reversed while those parallel to the
mirror are not reversed. For instance, if we hold a magnet
with its north end pointing from us and towards a mirror,
the optical image of it will have its North end pointing
towards us (fig. 6). But in the case of a magnetic image
this is inversed: the magnetic image has its South end point-
ing towards us. In fact the reflexion of a North end is a
South end. If, however, we look at the reflexion of the cur-
rents flowing in the solenoid instead of at the sign of the pole
of the solenoid, we see that laws of the optical case hold good
for the magnetic case. The image (fig. 7) of a current flowing
in any direction in a plane parallel to the mirror is another
current flowing parallel to the first and towards the same part.

A North pole viewed from the front is equivalent to a lævo-cheiral circulation of current. The image of this pole turned
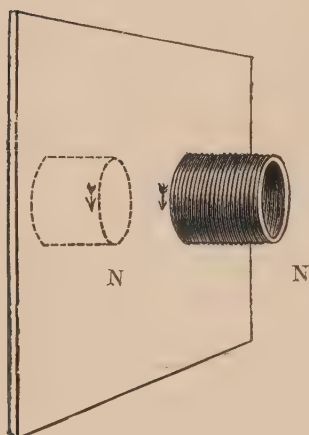
Fig. 7.   Fig. 8.
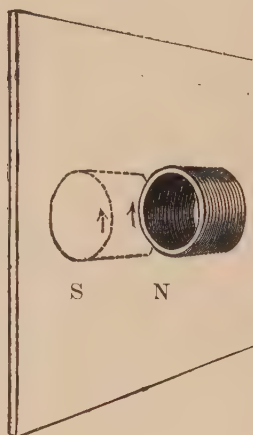
towards the mirror is, to the spectator, a dextro-cheiral circulation, or a South pole. The same is true if, as in fig. 8, the solenoid has its axis parallel to the plane of the mirror.

Fig. 9.   Fig. 10.

In order to try some effects with an isolated pole, a solenoid, 2 metres long and 1·5 centim. mean diameter, was uniformly wound with twelve turns per centimetre of a suitable wire. When a current of 15 amperes was passed through this a

pole of a strength of about 40 units (C.G.S.) was obtained,
and this might be regarded as an isolated pole at distances
not greater than 30 centim. from it. It was found that
bringing an iron plate within a few centimetres of this pole
produced very nearly the same effect upon the field as would
have been produced by bringing up a pole of opposite sign to
a point at double the distance.

The field close to the iron plate ought to have been exactly
twice as strong as if no iron plate were there. This was found
to be so within the limits of error of observation (about 3 per
cent.) when the pole was within 6 centim. of the iron plate.
As the distance was increased the throw of the needle became
too small to be accurately read, and the field being extremely
weak the susceptibility of the iron would be very low.

It was this weakness of the field (for one could not consider
the pole as a point except at distances exceeding 10 centim.)
that spoilt some experiments on kaleidoscope effects. One
can conceive that if the laws of reflexion hold good between
a magnetic pole and a sheet of iron, it ought to be possible, by
means of several sheets of iron, to build up a magnetic kaleido-
scope by which the effect of a great number of magnetic poles
could be produced from a single pole. The idea of a mag-
netic kaleidoscope was suggested by Lord Kelvin many years
ago, but it is doubtful whether it has ever been successfully
carried out. If, for instance, two plates of susceptible
material are inclined to each other at
an angle of 60 degrees, and the North
pole of a magnet placed evenly between
them, one would expect to get between
the plates exactly the same distribution
of field as if there were six poles, three
norths and three souths, spaced alter-
nately and evenly round the line of
intersection of the plates. It may,
however, be said to those who would
care to try this experiment, that they
must use a point pole of much greater
strength than 40 units, or plates much
more susceptible than iron if they hope to be at all successful.

Fig. 11.

The close connexion between the effect of an iron plate
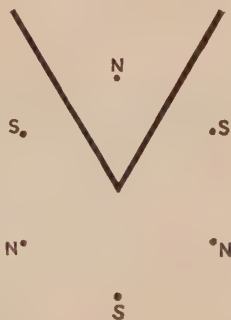
magnetically and that of a mirror optically naturally makes
us ask what would be the effect of a curved plate ?   Would it
give magnified or diminished images like a concave or convex
mirror, and what would be the position of these images with
regard to the object ?

If we take the case of spherical curvature, and consider the
iron as being infinitely susceptible, the answer to these
questions can be determined theoretically from the laws of
magnetic potential.   All the formulæ relating to electric
images already known are applicable to magnetic images.

If we consider two points A and B at which are placed
two poles respectively, of the strengths $+ m_1$ and $- m_2$, the

Fig. 12.
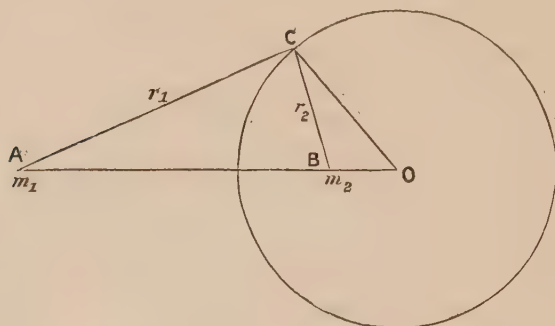
magnetic potential at a point C due to m_1 is $\dfrac{m_1}{r_1}$ where $r_1 = $ AC,

and that due to $- m_2$ is $- \dfrac{m_2}{r_2}$ where $r_2 = $ BC.

The total potential at C is therefore $\dfrac{m_1}{r_1} - \dfrac{m_2}{r_2}$.

If we choose C at a point of zero potential, we have

$$\frac{m_1}{r_1} - \frac{m_2}{r_2} = 0;$$

that is to say

$$\frac{m_1}{m_2} = \frac{r_1}{r_2}:$$

but $\dfrac{m_1}{m_2}$ is a constant; therefore, if we take C at the points of

zero potential $\frac{r_1}{r_2}$ will always be constant. But if a point

moves so that $\frac{r_1}{r_2}$ remains constant it moves on the surface of

a sphere, therefore the equipotential surface of value zero is in the form of a spherical shell. If O be the centre of the sphere, it follows from a well-known property of a circle that

$$\frac{AO}{OC} = \frac{r_1}{r_2} = \frac{m_1}{m_2}.$$

If, therefore, we are given m_1 we can find from the equation

$$m_2 = m_1 \frac{OC}{AO},$$

the strength of pole m_2 which when placed at B will give zero potential on a given spherical shell.

Now consider a magnet pole $+m_1$ (fig. 13) brought up to

Fig. 13.
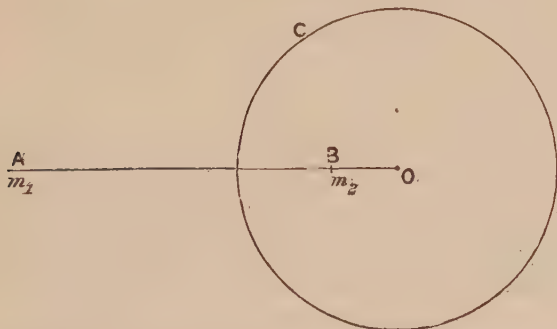
a point A near a sphere of very susceptible material whose radius is large as compared with the distance between A and its surface, so that we may neglect the potential of the sphere due to m_1 in dealing with the potential of points in the field we are considering, and so that we may take the surface of the sphere as an equipotential surface of practically zero value *. The distribution of the field outside the sphere will

* [*Note added after reading of Paper.*—At this point our original paper contained the following remark as a footnote :—" If the magnetic object is large, or is far removed, then besides the image as above defined it is

be the same as if no sphere were there, but instead a pole of strength $-m_1\dfrac{OC}{AO}$ were placed at B.   We may say that the sphere acting like a convex mirror has given a diminished image at B.

We can find the position of B by the following construction :—Describe the arc OC (fig. 14) with A as a centre, and then describe the arc OB with C as a centre, B being on the

Fig. 14.

necessary to take into account the raising of the potential of the whole sphere, which would be represented by another image at the centre." The case is then analogous to the case of an electric charge q brought up to an insulated conducting sphere having no previous charge. Lord Kelvin, in a paper dated July 7th, 1848, has shown that the effect on external points of the charge residing on the surface of such a sphere is the same as the effect of a charge $-\dfrac{r}{d}q$ at B (fig. 14), and a further charge of $+\dfrac{r}{d}q$ at the centre O.   His reasoning being applicable to the magnetic case, we see that the image of a North pole at A consists of a doublet having a South pole situated at B of strength $m_1\dfrac{r}{d}$ and a North pole also of the strength $m_1\dfrac{r}{d}$ situated at the centre O.   Now any magnetic object such as a solenoid will have a South pole as well as a North pole, and if the object is small compared with the size of the sphere, both North and South may be taken as equidistant from the centre, and their images at the centre will therefore neutralize each other, and we have left the image that is considered in the text.   If the object is large as compared with the size of the sphere, then in both the magnetic case and the optical case there is a confusion of images.]

line AO. The condition that $\frac{r_1}{r_2}$ shall be constant for all points on the circumference is that $\frac{OC}{BO}$ shall be equal to $\frac{AO}{OC}$, and this is seen to be true from the similarity of the triangles ACO and COB.

If we had any number of magnetic points outside the sphere each would have its virtual image inside the sphere; thus any form of magnet, such as a solenoid carrying a current, would also have its image.

It is easy to see that the experiments above mentioned with a large iron plate can be explained in this way. For if we may consider the plate as part of an infinite sphere:

Fig. 15.
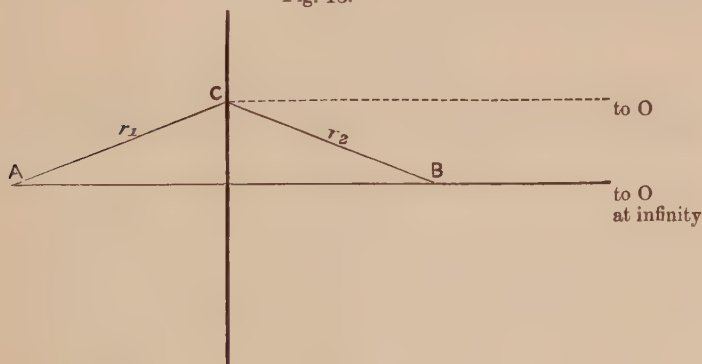
in this case $\frac{AO}{OC} = 1$; therefore $\frac{m_1}{m_2} = 1$; therefore $m_1 = m_2$:

and further $\frac{r_1}{r_2} = 1$; therefore $r_1 = r_2$.

To assimilate to optical formulæ let us now express the relations in terms of the distance of object and image respectively from the pole E of the mirror, and write $AO = d$; $BO = b$; $u = AE = d - r$; $v = BE = r - b$ (fig. 14). Now

$$\frac{d}{r} = \frac{r}{b}, \text{ by similar triangles.}$$

This may be written

$$\frac{u + r}{r} = \frac{r}{r - v};$$

whence

$$\frac{1}{u} + \frac{1}{r} = \frac{1}{v}.$$

The well-known optical mirror formula for a spherical mirror differs from this in having $\frac{2}{r}$ instead of $\frac{1}{r}$.

In order that a magnetic image as at B in fig. 14 should be produced by a magnetic object at A, and with the relative strengths m_2 and m_1, not only must the reflecting surface pass through the point E such that

$$\frac{AE}{BE} = \frac{m_1}{m_2},$$

but the curvature of the magnetic mirror is defined by the condition previously laid down that

$$\frac{AO}{CO} = \frac{m_1}{m_2} \text{ also.}$$

Now the optical mirror which will give at B the image of A, and have its middle-point situated at E (as defined by the

Fig. 16.
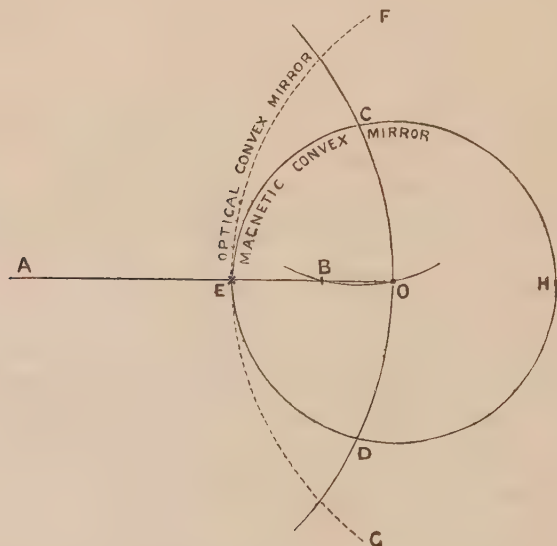
former of these conditions) will not have O for its centre. In fact it will be a spherical surface having half the curvature or twice the radius. If (fig. 16) the dotted circle FEG be

drawn with centre H, we have the sectional view of the optical convex mirror which will give the same position of image as the magnetic convex mirror shown by the circle CED.

In conclusion, it may be remarked that the construction of fig. 14 affords a new and simple graphic method of finding the position of optical images, by the device of using the auxiliary circle whose diameter is the radius of the spherical surface.

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XXX. *A Modification of the Ballistic-Galvanometer Method of Determining the Electromagnetic Capacity of a Condenser.* *By* F. WOMACK[*].

THE method consists in placing the condenser in parallel with one arm S of a Wheatstone-bridge arrangement of non-inductive resistances. A balance for steady currents having been obtained, the condenser is placed in circuit, and the throw θ determined due to the depression of the battery-key. The condenser is then thrown out of circuit, and the proportionality of the arms of the bridge disturbed by changing the value of S. The steady deflexion α due to this want of balance is read. From these two readings and the known values of S, the capacity is immediately determined.
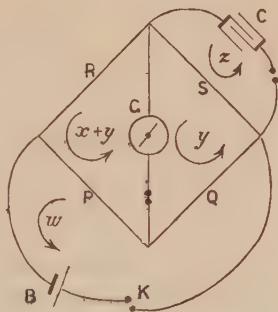
If x, y, z, w denote the integral cyclic currents reckoned from the instant of make,

$$
\begin{aligned}
P(x+y-w) + \quad Gx \quad + R(x+y) &= 0 \\
-Gx \quad + Q(y-w) + S(y-z) &= 0 \\
P(w-x-y) + \quad Bw \quad + Q(w-y) &= \int E \cdot dt \\
z &= CS\dot{y}_\infty
\end{aligned}
$$

Substituting for z in the second equation and rearranging,

$$
\left.
\begin{aligned}
(P+G+R)x + (P+R)y - \quad Pw \quad &= 0 \\
-Gx \quad + (Q+S)y - \quad Qw \quad &= CS^2\dot{y}_\infty \\
-Px \quad - (P+Q)y + (P+Q+B)w &= \int E \cdot dt
\end{aligned}
\right\}
\begin{aligned}
&\text{(i.)} \\
&\text{(ii.)} \\
&\text{(iii.)}
\end{aligned}
$$

Hence

$$x = \frac{\begin{vmatrix} 0 & P+R & -P \\ CS^2\dot{y}_\infty & Q+S & -Q \\ \int E\,.\,dt & -(P+Q) & P+Q+B \end{vmatrix}}{\begin{vmatrix} P+G+R & P+R & -P \\ -G & Q+S & -Q \\ -P & -(P+Q) & P+Q+B \end{vmatrix}} = \frac{CS^2\dot{y}_\infty \begin{vmatrix} -(P+Q) & P+Q+B \\ P+R & -P \end{vmatrix}}{\Delta}$$

$$= \frac{CS^2\dot{y}_\infty}{\Delta}(PB+PR+QR+BR).$$

To find the steady current $\dot{y}_\infty$ ultimately flowing through S we have the same equations, except that the right-hand side of (ii.) is zero, and $\dot{x}_\infty$, $\dot{y}_\infty$, $\dot{w}_\infty$ instead of x, y, w. Hence

$$\dot{y}_\infty = \frac{\begin{vmatrix} P+G+R & 0 & -P \\ -G & 0 & -Q \\ -P & E & P+Q+B \end{vmatrix}}{\Delta} = \frac{E}{\Delta}(PQ+GQ+RQ+PG).$$

Therefore the integral current through the galvanometer is

$$x = \frac{CS^2E}{\Delta^2}(PB+PR+QR+BR)(PQ+GQ+RQ+PG).$$

When the balance is disturbed by S becoming $S+dS$, the cyclic equations for steady currents are

$$\left.\begin{array}{l} (P+G+R)\dot{x}_\infty + (P+R)\dot{y}_\infty - P\dot{w}_\infty = 0 \\ -G\dot{x}_\infty + (Q+S+dS)\dot{y}_\infty - Q\dot{w}_\infty = 0 \\ -P\dot{x}_\infty - (P+Q)\dot{y}_\infty + (P+Q+B)\dot{w}_\infty = E \end{array}\right\},$$

which give

$$\dot{x}_\infty = \frac{\begin{vmatrix} 0 & P+R & -P \\ 0 & Q+S+dS & -Q \\ E & -(P+Q) & P+Q+B \end{vmatrix}}{\Delta \text{ nearly}} = \frac{EP}{\Delta \text{ nearly}}dS,$$

assuming the value of the determinant Δ to be but little affected by the change of S.

PHYSICAL SOCIETY OF LONDON.

ABSTRACTS OF PHYSICAL PAPERS

FROM FOREIGN SOURCES.

APRIL 1895.

GENERAL PHYSICS.

274. *Volume-Changes on Neutralising Dilute Solutions.*   **G. Tammann.** (Zschr. phys. Chem. 16. pp. 91–96, 1895.)—The investigation was undertaken to ascertain the influence of concentration and temperature on the volume-changes studied by Ostwald (J. f. prakt: Chem. 18. p. 328, 1876) and Ruppin (Zschr. phys. Chem. 14. p. 467, 1894). Kohlrausch and Hallwachs' results (Gottinger Nachrichten, p. 350, 1893) were used to test the accuracy of the Sprengel method (Pogg. Ann. 150. p. 459, 1873) for determining the specific gravities. The densities of weak solutions of $NaOH$, HCl, $\frac{1}{2}H_2SO_4$, $CH_2ClCOOH$, CH_3COOH, $NaCl$, $CH_2ClCOONa$, $CH_3 . COONa$, Na_2SO_4 are given in the paper. The results show that the alterations of volume on the neutralisation of caustic soda with the strong dissociated acids are only slight with increasing dilution, but increase with a concentration of 1·00–0·005 gramme-equivalents per litre to about 20 per. cent., with sulphuric acid 90 per cent., with monochloracetic acid 60 per cent., and acetic acid 50 per cent.   S. R.

275. *Changes of Volume accompanying Neutralisation.*   **G. Tammann.** (Zschr. phys. Chem. 16. pp. 139–146, 1895.)— The changes of volume are regarded as being due to changes in the internal pressure of the solutions. These changes of internal pressure produce practically the same change in the volume of the solution as an equal change of external pressure would produce in the volume of the pure solvent under a pressure

of ΔK atmospheres. The change of volume of the dissolved substance is regarded as negligible for fairly dilute solutions. The internal pressure of the solution exceeds that of the pure solvent by ΔK atmos; the pure solvent when exposed to a pressure of ΔK atmos is therefore regarded as being in a condition comparable with that of the solution. With the help of Amagat's determinations of the compressibility of water, and of the values of ΔK obtained in previous communications (Zschr. phys. Chem. 13. p. 178; 14. p. 167), the changes of volume which should occur on mixing solutions of KOH or NaOH with HCl are calculated, and found to be in fairly good agreement with those determined. Further, as the values of ΔK are nearly independent of the temperature, the changes of volume which accompany neutralisation at different temperatures should be proportional to the values of the coefficient of compressibility of water at those temperatures. According to Amagat the coefficient of compressibility of water decreases from 0° to 50°, and then increases again. The changes of volume on mixing solutions of NaOH and of HCl follow a similar course.   T. E.

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276. *Specific Heat of Gases.* **H. Petrini.** (Zschr. phys. Chem. 16. pp. 97–117, 1895.)—Basing themselves on more or less reasonable assumptions with regard to the constitution of the molecules of gases, certain physicists have obtained expressions which give the ratio $c_p : c_v$; c_p being the specific heat at constant pressure, and c_v the specific heat at constant volume. The author deals specially with the equation

$$c_p/c_v = 1 + 2/(2n+1), \quad \cdots \cdots (1)$$

a relation which is obtained from an equation due to Boltzmann by assuming that in the case of a molecule containing n atoms one can move freely in all directions, but the remaining $(n-1)$ can only move so as to remain on the surfaces of spheres which have the unique atom as centre; by means of (1), and other equations thence derived, the author studies the results which have been obtained by various experimentalists with the gases the symbols for whose molecules are generally taken as Hg (monatomic), Hg, N_2, O_2, CO, NO, HCl, HBr, HI, Cl_2, Br_2, I_2, ClI (diatomic), H_2O, H_2S, CO_2, N_2O, NO_2 or N_2O_4, SO_2, CS_2 (triatomic), P_4, NH_3 (4 atoms to the molecule), CH_4 (5 atoms), C_2H_4 (6), PCl_3, $AsCl_3$ (4), $SiCl_4$, $TiCl_4$, $SnCl_4$, $CHCl_3$, CH_2Cl_2 (5), $C_2H_4Cl_2$, $C_2H_3Cl_3$ (8), CH_3Cl (5), CH_4O (6), C_2H_4O (7), C_2H_5Cl, C_2H_5Br (8), C_2H_6O (methyl-ether), C_2H_6O (alcohol), C_2H_5CN (9), C_3H_6O (10), C_6H_6 (12), $C_3H_8O_2$ (13), $C_4H_8O_2$ (14), $C_4H_{10}S$, $C_4H_{10}O$ (15), and $C_{10}H_{16}$ (26). The experimental determinations *agree* with theory in the case of Hg; *likewise* for H_2, N_2, O_2, CO, NO, HCl, HBr, HI; but there is *disagreement* in the case of Cl_2, Br_2, I_2, and ClI. The agreement is fairly satisfactory with H_2O, H_2S, CO_2, and N_2O; dissociation complicates matters in the case of N_2O or N_2O_4; CS_2 gives only

approximate agreement. To suit equation (1) the molecules of carbon disulphide and phosphorus must be considered as containing 5 molecules. The results given by NH_3, CH_4, C_2H_4, PCl_3, $AsCl_3$, $SiCl_4$, $TiCl_4$, $SnCl_4$, $CHCl_3$, CH_4Cl_3, and $C_2H_3Cl_3$ show *no agreement* with theory; the *agreement* is satisfactory with the remaining gases. The author concludes that equation (1) holds true with certain exceptions, and, using it as a basis, shows mathematically how to find the degree of dissociation of a collection of molecules and atoms. Finally there is given a very complete table. A. G.

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277. *Solution of Solids in Vapours.* **P. Villard.** (C. R. 120. pp. 182–184, 1895.)—The author refers to the work of Pictet (C. R. 120. p. 64), Hannay and Hogarth (P. R. S. 30. p. 178), and his own on the solution of iodine in CO_2 (J. de Phys. [3] 3. Oct. 1894), and points out that they are in accord. S. R.

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278. *Elastic Hysteresis.* **M. Cantone.** (Rend. Acc. Lincei, 3. 1894.) [Abstract by A. Rovida in N. Cim. (4) 1. pp. 21–25, Jan. 1895.]—As the result of experiments (no details given) the author formulates the following results—which, however, are mostly not new. When the stress on a body is continuously increased or decreased, its elasticity changes continuously, even though the stress pass from positive to negative or *vice versa*; but on changing the sense of variation of the stress, the elasticity changes abruptly. On performing a cycle of changes of stress, if the maximum previous stress is not overpassed, the body returns to its initial state : a closed curve is described on the stress-strain diagram, and there is dissipation of energy. Small oscillations or shaking produce considerable effect, and may destroy altogether the permanent deformation. Repeated treatment affects different substances differently, augmenting the "area of hysteresis" in the case of nickel, diminishing it in that of brass. Damping of vibrations may be attributed to the dissipation of energy involved in executing elastic cycles such as were observed. R. A. L.

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279. *Volume of Substances in Solution.* **J. Traube.** (Zschr. an. Chem. 8. pp. 12–76, 1895.)—The quantity which remains after subtracting from that volume of a solution which contains a gram-molecule of the dissolved substance the volume of the water contained in the solution, does not represent the volume of the salt in the solution, as it is sometimes negative. The author, however, proposes to call it the atomic or molecular solution-volume of the dissolved substance. He calculates the values of this quantity for a great many compounds of some 50 different elements, partly from original determinations of the specific gravity of the solutions, partly from determinations by other authors. The following results extracted from the extensive memoir will serve to indicate the nature of its contents :—Compounds of H, Li, Na, Ag (Cu, Au, Hg, monovalent) with the same acid radicals possess very nearly

the same molecular solution-volume. For the group of alkali metals, the following mean differences between the volumes of their salts with the same acids were found :—

K — Na.	NH$_4$ — K.	Rb — K.	Cs — Rb.
10	9·2	9·7	11·0

Similar results were obtained with other metals. The author sums up thus :—The influence which a given ion exerts on the value of the molecular solution-volume is (assuming that the dissolved substances are ionised to the same extent) independent, or nearly so, of the other ion which is present in the solution. The mean difference found between a bromide and a chloride of the same metal was 7·5. With non-electrolytes, however, a much smaller difference was found: for example, $PBr_3 — PCl_3 = 2$. Similar differences were found in the case of other non-electrolytes. With rising temperature the molecular solution-volume increases at first, reaches a maximum [frequently between 50 and 70], and then decreases. The differences between the molecular volumes of the solid salts were in many cases nearly equal to the differences between the molecular solution-volumes. T. E.

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280. *Meteorograph for the Mont Blanc Observatory.* **J. Janssen.** (Ann. Bur. Long. C 1–C 5, 1895.)—Richard has constructed a meteorograph which, once wound up, will keep going the whole winter and spring. The actuating weight of 90 kg. descends about 6 m. in the course of eight months. The clockwork is provided with a Denison escapement which, in an atmosphere free of dust, hardly requires any oil. According to Denison, no variation in the amplitude has been observed, even with the oil frozen and the consistency of tallow. The main arbor turns once in 24 hours, and unrolls the various paper bands with different velocities. The barometer is a Gay-Lussac instrument, with a very large cistern; Janssen decidedly prefers a mercury barometer. The temperature is determined by means of a Bourdon tube, which, as well as the Saussure hair-hygrometer, is placed outside, while the registering devices are within the building. The direction and velocity of the wind are marked on the same sheet of paper. A vane or set of Robinson's cups turns a cylinder on which a number of cams are arranged in a spiral ; these cams raise pens, one after the other, which trace their curves as long as their cams are in action. Eight styles indicate the direction of the wind, ten the velocity. One revolution of the cylinder represents 10 km., and each pen is in gear for one tenth of a revolution. The velocity is thus indicated by the length of the mark. This anemometer will be placed in a special chamber of the hall, to protect the apparatus from the dust made by the men at work. The meteorograph could not be put up in the summer of 1894, owing to the rough weather. Though improvements may prove necessary afterwards, in the meantime maximum and minimum thermometers, and instruments to test the penetration of solar atmospheric heat into the snow, have been put up at the

Observatory, the Grands-Mulets, and elsewhere. Large-scale photographs have also been taken with a Meudon instrument, in order to ascertain whether there are any vibrations on the summit; as yet, none have been observed. Whymper spent some time last summer at the observatory, and found the gallery partially obstructed. Thierry encountered still greater obstacles. The paper does not enter into any details. ·   H. B.

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281. *Schuller's Automatic Air-Pump.* **K. Kiss.** (Zschr. Instrumk. 15. pp. 59–72, Feb. 1895.)—Schuller combines a water- and a mercury-pump to an automatically acting instrument which can be obtained from the Glastechnical Laboratory, Budapest. Through the central neck of a three-necked Woulf bottle enters a glass tube, reaching almost to the bottom and enlarging above into a sphere, surmounted by a vertical tube with two valves. Below the sphere a barometer-tube branches off, fitted with three gauges and a drying-tube, through which connection is made with the body to be evacuated. The third neck contains a three-way cock, communicating either with the atmosphere or with a T-piece making connection with the water-pump and with accessory parts. $CaCl_2$ and P_2O_5 tubes are always inserted. A U-branch of the water-pump contains a smaller regulator glass sphere, which is suspended and counterbalanced by a weight in such a way that it will descend to a certain level when filled with mercury from below, and be raised by the weight, when part of the mercury has flown off above through a rubber tube of 50 cm. length connecting it with the first neck of the Woulf bottle. The string by which the weight is suspended is continued, and joined to the arc of a lever which turns the three-way cock. In this manner the water-pump is thrown in and out of action with the help of this regulator. The stroke takes from two to three minutes, the ports being narrow to avoid violent rushes; the air-passage of the three-way cock is only 0·25 mm. in width. The rubber tubes are not in any contact with the Torricelli vacuum. The accessory parts are apparatus for producing Geissler vacua and filling glasses for spectroscopic researches, and retorts or tubes for distillation, in which the distilling may go on for several days without special supervision. The valves are triangular glass plates, the seals mercury-columns of 2 or 3 mm. height and width. Kiss experimented with two pumps of his own make; he does not employ any oil or grease. In the Woulf bottle he keeps a few dishes floating filled with P_2O_5. His manometers are of the Arago-McLeod type. To produce a vacuum of $0·5 \times 10^{-4}$ mm. in a receiver of 223·34 c.c. capacity, the pump itself adding 1111·14 c.c., required 57 strokes of 2 min. 4 sec. each. At first the evacuation proceeded with great regularity. When a vacuum of some 10^{-4} mm had been attained, however, the pressure rose sometimes after a new stroke by one or two of these units. The most striking case was after the 43rd stroke. The pump had been left standing overnight, and, after the 44th stroke, the pressure rose from 1 to 12×10^{-4} mm.

The author does not think that there was any leakage, but **considers** that air is withheld by mercury and given **off afterwards by** friction. Mercury boiled four times in barometer-tubes for several hours was not found quite free of air. A Geissler tube forming part of the receiver first shone in violet light. Then green appeared in the centre of the tube; the green spread, and both the violet and the green grew fainter. At 13×10^{-3} mm. sparks of 45 mm, length appeared on the outside of the tube; at a pressure 10 and 5×10^{-3} mm., the tube was dark or occasionally spotted with green, and sparks of 65 mm. length were observed. The Geissler tube was several times heated during the course of the experiments. Kiss also heats the glass tubes of the pump. Nothing is said about any electric phenomena at the extreme exhaustion realised, 0·00005 mm.   H. B.

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282. *Solubility of Salts in Organic Liquids.* **C. E. Linebarger.** (Am. J. 49. pp. 48–53, Jan. 1895.)—From thermodynamical considerations Schroeder deduced the law that the solubilities at equal intervals from the temperature of fusion for different solid bodies and in different solvents are the same. The law was tested by experiments on organic substances dissolved in organic solvents. Le Chatelier developed a similar law, and tested it by studying the solubility of NaCl in fused Na_2CO_3 and $BaCl_2$; also with Li_2SO_4 in fused $CaSO_4$, Li_2CO_3, and Na_2SO_4. The object of the paper is to discuss in the light of this law the data we possess on the solubility of inorganic salts in normal organic liquids. The greater part of the data already obtained are for alcohols, which appear from recent experiments to be made up of associated molecules. Some data by Étard on the solubility of mercuric and cupric chlorides in ethereal salts, and those by Arctowski on the mercuric halides in carbon bisulphide, are examined. To these the author adds results on the solubilities of cadmium iodide, mercuric chloride, and silver nitrate in benzene and ethyl ether. None of these cases agree with the law; and it is concluded that, although approximately true for the cases investigated by Schroeder and Le Chatelier, it is not applicable to the case of inorganic salts in non-associated organic solvents.   S. S.

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283. *Continuity of the Solid and Liquid States.* **W. Spring.** (Zschr. phys. Chem. 15. pp. 65–78, 1894.)—In a former paper the author showed that metals can be made to unite when strongly compressed. In the present paper he shows that similar union is obtained without compression by heating the metals in contact to a temperature below fusion. Cylinders of the metals have their ends made accurately plane, and are held together in a chamber, where they are heated for some hours to temperatures of from 200° to 400°. If both cylinders are of the same metal, it is found that aluminium, bismuth, cadmium, copper, gold, platinum, lead, and zinc weld together completely: antimony

unites imperfectly. A copper cylinder with a very shallow spiral groove cut on one end is placed with this end on a mica plate. After heating to 400° for 8 hours the groove disappears. If cylinders of two different metals are heated in contact, alloys are formed. Zinc and copper after 6 hours' heating penetrate 18 mm. into each other. The author considers the experiments point to continuity between the solid and liquid states. As the melting-point is approached, some of the molecules have velocities corresponding to a temperature above the melting-point, and so acquire the power of diffusing amongst the others. To prove the existence of sublimation, cylinders of copper and zinc are separated by a mica plate containing a hole. After heating the zinc is found to have sublimed through the hole and formed a yellow film of brass on the copper. J. W. C.

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284. *Equations of Elasticity.* **C. Somigliana.** Pavia.—I. If u, v, w be the components of displacement of a point in an elastic body, the conditions of equilibrium at a point within its substance are

$$k\mathrm{X} + (2\lambda + \mu)\frac{d\theta}{dx} + \mu\nabla^2 u = 0, \quad \ldots \ldots \quad (1)$$

with two corresponding equations in y and z, in which X, Y, Z are the components of external force acting at a point within the substance, k is the density, and $\theta = \dfrac{du}{dx} + \dfrac{dv}{dy} + \dfrac{dw}{dz}$, and λ, μ are constants depending on the nature of the substance. At a point on the surface the conditions are

$$\mathrm{L} + 2\mu\frac{d\mathrm{U}}{d\nu} + 2\lambda\theta\frac{dx}{d\nu} + \mu\frac{dy}{d\nu}\left(\frac{d\mathrm{V}}{dx} - \frac{d\mathrm{U}}{dy}\right) - \mu\frac{dz}{d\nu}\left(\frac{d\mathrm{U}}{dz} - \frac{d\mathrm{W}}{dx}\right) = 0, \quad (2)$$

with two corresponding equations. In this $d\nu$ is an element of the normal to the bounding surface measured inwards, and L, M, N are the components of external superficial force. Also U, V, W are the values of u, v, w on the surface. If X = Y = Z = 0 at all points within the substance, equilibrium may be maintained by the superficial forces L, M, N with a certain system of displacements.

Let, for instance, the displacements be

$$u = \frac{\alpha}{2}\frac{d^2 r}{dx^2} + \frac{1}{r}, \quad v = \frac{\alpha}{2}\frac{d^2 r}{dx\,dy}, \quad w = \frac{\alpha}{2}\frac{d^2 r}{dx\,dz},$$

in which r is the distance of the point $(x\,y\,z)$ in the substance at which u, v, w are the displacements from an external point $x'\,y'\,z'$ and α is a constant. In this case, since X = 0, (1) becomes

$$(2\lambda + \mu)\frac{\alpha}{2}\frac{d^2}{dx^2}\nabla^2 r + (2\lambda + \mu)\frac{d^2\frac{1}{r}}{dx^2} + \frac{\mu\alpha}{2}\frac{d}{dx}\nabla^2\frac{dr}{dx} = 0,$$

which is satisfied if

$$\alpha = -\frac{2\lambda+\mu}{2\lambda+2\mu}.$$

Using the value of α thus obtained, we obtain for the superficial equations,

$$L' = -2\alpha\mu\left\{\frac{d}{d\nu}\frac{d^2\frac{r}{2}}{dx^2}-\frac{dx}{d\nu}\frac{d\frac{1}{r}}{dx}\right\}-\mu\frac{d\frac{1}{r}}{d\nu},$$

$$M' = -2\alpha\mu\left\{\frac{d}{d\nu}\frac{d^2\frac{r}{2}}{dx\,dy}-\frac{dy}{d\nu}\frac{d\frac{1}{r}}{dx}\right\}-\mu\left\{\frac{dx}{d\nu}\frac{d\frac{1}{r}}{dy}-\frac{dy}{d\nu}\frac{d\frac{1}{r}}{dx}\right\},$$

$$N' = \&c.,$$

thus giving the component forces L′, M′, N′ necessary to maintain the body in equilibrium when it has the deformation denoted by u, v, w.

Our author now makes use of a theorem due to Betti, according to which, if u, v, w be displacements when the body is in equilibrium under the forces X, Y, Z within the mass, and L, M, N superficial, and u', v', w' the displacements when the forces are X′ &c.; then, denoting the superficial values of u, v, &c. by U, V, &c., we have

$$\iiint(Xu'+Yv'+Zw')\,dx\,dy\,dz+\iint(LU'+MV'+NW')\,dS$$
$$= \iiint(X'u+Y'v+Z'w)\,dx\,dy\,dz+\iint(L'U+M'V+N'W)\,dS.$$

To apply this theorem, let u', v', w' be the displacements already stated (which might be called auxiliary displacements), $X'=Y'=Z'=0$, L′, M′, N the superficial forces above given corresponding to u', v', w'. The forces X, Y, Z, L, M, N being given, the integrals $\iiint Xu'\,dx\,dy\,dz$ &c. and $\iint LU'\,dS$ &c. are known. And the equation gives us u, v, w, the displacements which the body will assume under the action of those forces, in terms of the forces and of the superficial values of u, v, w in the form

$$u(x\,y\,z)=\frac{1}{4\pi\mu}\{\rho\int(Xu'+Yv'+Zw')\,dx\,dy\,dz\}$$
$$+\iint(LU'+MV'+NW')\,dS$$
$$+\iint(L'U+M'N+N'W)\,dS.$$

The same formula is shown to hold for a point on the surface, provided we write $2\pi\mu$ for $4\pi\mu$. It is further shown that every deformation of a homogeneous isotropic elastic body can be resolved into three deformations, depending on (1) the forces X, Y, Z acting within the substance, (2) the superficial forces L, M, N, (3) the superficial displacements U, V, W.

In II. Somigliana obtains, where the displacements are continuous, other expressions for u, v, w in terms of the forces X, Y, Z,

the superficial forces L, M, N, and the superficial displacements U, V, W, as follows. Integrating by parts, we find

$$\iiint \frac{1}{r}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right) dx\, dy\, dz$$

$$= -\iint \frac{1}{r}\left(U\frac{dx}{d\nu}+V\frac{dy}{d\nu}+W\frac{dz}{d\nu}\right) dS$$

$$+\iiint \left(u\frac{d}{dx'}+\frac{d}{dy'}+w\frac{d}{dz'}\right)\frac{1}{r} dx\, dy\, dz,$$

$$\iiint \frac{1}{r}\left(\frac{dw}{dy}-\frac{dv}{dz}\right) dx\, dy\, dz = -\iint \left(W\frac{dy}{d\nu}-V\frac{dz}{d\nu}\right)\frac{1}{r} dS$$

$$+\iiint \left(w\frac{d}{dy}-v\frac{d}{dz}\right)\frac{1}{r} dx\, dy\, dz,$$

with two other equations corresponding to the last. From which he derives the following :—

$$4\pi u = (2\lambda+\mu)\frac{d}{dx'}\iiint \frac{1}{r}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right) dx\, dy\, dz$$

$$+k\iiint \frac{X}{r} dx\, dy\, dz + \iint L_1\frac{1}{r} dS$$

$$+\mu\iint \left\{ U\frac{d}{d\nu}\frac{1}{r}+V\left(\frac{dx}{d\nu}\frac{d}{dy}-\frac{dy}{d\nu}\frac{d}{dx}\right)\frac{1}{r}\right.$$

$$\left.-W\left(\frac{dz}{d\nu}\frac{d}{dx}-\frac{dx}{d\nu}\frac{d}{dz}\right)\frac{1}{r}\right\} dS,$$

with two corresponding equations. And as in the former case, we have three systems of displacements separately satisfying the conditions of equilibrium, having regard to (1) the internal forces X, Y, Z, (2) the superficial forces L, M, N, (3) the superficial displacements U, V, W.

In III. is given the expansion of r, or

$$\sqrt{(x-x')^2+(y-y')^2+(z-z')^2},$$

in a series of Legendre's coefficients. If, namely,

$$r^2 = \rho^2+\rho'^2+2\rho\rho'\mu,$$

in which $\rho'<\rho$, and we assume

$$r = \sum_{n=0}^{n=\infty} \frac{\rho'^n}{\rho^{n-1}} P'_n,$$

we have, since $\dfrac{dr^2}{d\mu} = 2r\dfrac{dr}{d\mu} = -2\rho\rho'$,

$$\frac{1}{r} = -\sum_{n=0}^{n=\infty} \frac{\rho'^{-1}}{\rho^{n_a}} \frac{dP'_n}{d\mu}.$$

But in Legendre's series

$$\frac{1}{r} = \sum_{n=0}^{n=\infty} \frac{\rho'^n}{\rho^{n+1}} \, \mathrm{P}_n \, ;$$

whence, comparing the two series, we find

$$\mathrm{P'}_n = \frac{1}{2n-1} \, (\mathrm{P}_n - \mathrm{P}_{n-2})$$

and

$$r = \sum_{n=0}^{n=\infty} \left(\frac{\rho'^2}{2n+3} - \frac{\rho^2}{2n-1} \right) \frac{\rho'^n}{\rho^{n+1}} \, \mathrm{P}_n.$$

Hence, further, it is found that the most general form of ω, a rational integral homogeneous function of x, y, z of degree n which satisfies the equation $\Delta_2 \Delta_2 \omega = 0$ is $\rho^n(\mathrm{Y}_n + \mathrm{Y}_{n-2})$, where Y_n and Y_{n-2} are two spherical surface harmonics. The corresponding function of degree $-(n-1)$ is $\frac{1}{\rho^{n-1}} (\mathrm{Y}_n + \mathrm{Y}_{n-2})$.

If now, in the equations given in I. and II. for determining u &c., we expand r and $\frac{1}{r}$ in series according to the foregoing, the terms corresponding to any assigned value of n constitute a system of displacements satisfying the conditions of equilibrium. Hence he deduces the following theorem :—" If about any point $a\,b\,c$ taken in the interior of the body we describe a sphere of radius less than the least distance between $a\,b\,c$ and any point on the surface, every possible deformation which can exist with zero forces within the substance can be represented within that sphere by the sum of an infinite series of deformations whose displacements at $x\,y\,z$ are rational integral homogeneous functions of $x-a, y-b, z-c$. The coefficient in these functions can be expressed in terms of the superficial forces and displacements."

In IV. our author considers, instead of the $x\,y\,z$ of ordinary space, a system of n variables $x_1 \ldots x_n$ (where $n > 2$) representing space as we may know it in a future case of existence. In this case, if Δ_2 denote $\sum_1^n \frac{d^2}{dx^2}$, and

$$r^2 = \Sigma \{ (x_1 - x_1')^2 + (x_2 - x_2')^2 + \ldots + (x_n - x_n')^2 \},$$

we get

$$\Delta_2 \frac{1}{r^{n-2}} = 0$$

instead of the ordinary

$$\Delta_2 \frac{1}{r} = 0 \, ;$$

also

$$\Delta_2 \left(-\frac{r^{-n+4}}{2(n-4)} \right) = \frac{1}{r^{n-2}}.$$

And he finds that on certain hypotheses equations analogous to

those above treated hold good, and lead to analogous solutions, viz. :—

$$u_1' = a\,\frac{d^2V}{dx_1^{\,2}} + \frac{1}{r^{n-2}}.$$

$$u_2' = a\,\frac{d^2V}{dx_1\,dy_2},$$

&c.

S. H. B.

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285. *Integration of the Equations of Equilibrium in Isotropic Elastic Bodies.* **C. Somigliana.** (N. Cim. 3. 36, pp. 4–28, 1894.)— I. The equations of equilibrium for the interior of the body may be put in the form

$$a^2\frac{d\theta}{dx} + b^2\left(\frac{dq}{dz} - \frac{dr}{dy}\right) + kX = 0, \quad \ldots \ldots \quad (1)$$

with two corresponding equations in which

$$\theta = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz},$$

u, v, w being the displacements of a point in the substance, and

$$p = \frac{dw}{dy} - \frac{dv}{dz}, \quad q = \frac{du}{dz} - \frac{dw}{dx}, \quad r = \frac{dv}{dx} - \frac{du}{dy}, \text{ and } k \text{ is the density.}$$

Let ϕ, ψ, χ be three new functions, which he calls *generating functions*, such that

$$u = b^2\frac{d}{dx}\left(\frac{d\phi}{dx} + \frac{d\psi}{dy} + \frac{d\chi}{dz}\right)$$
$$+ a^2\left\{\frac{d}{dz}\left(\frac{d\phi}{dz} - \frac{d\chi}{dx}\right) - \frac{d}{dy}\left(\frac{d\psi}{dx} - \frac{d\phi}{dy}\right)\right\}, \quad \ldots \quad (2)$$

$$v = \&c.$$

Making these substitutions for u, v, w in equations (1), we reduce them to

$$\left.\begin{array}{l} a^2b^2\Delta_2\Delta_2\phi + kX = 0, \\ a^2b^2\Delta_2\Delta_2\psi + kY = 0, \\ a^2b^2\Delta_2\Delta_2\chi + kZ = 0, \end{array}\right\} \quad \ldots \quad \ldots \quad (3)$$

II. Now if $X = Y = Z = 0$ these equations are satisfied by any one of the three systems

$$\phi = r, \quad \psi = 0, \quad \chi = 0,$$
$$\phi = 0, \quad \psi = r, \quad \chi = 0,$$
$$\phi = 0, \quad \psi = 0, \quad \chi = r,$$

and values of u, v, w satisfying the first of these are

$$u_1 = (b^2 - a^2)\frac{d^2\rho}{d\xi^2} + \frac{2a^2}{r},$$

$$v_1 = (b^2 - a^2)\frac{d^2\rho}{d\xi\, d\eta},$$

$$w_1 = (b^2 - a^2)\frac{d^2\rho}{d\xi\, d\zeta},$$

with
$$\rho^2 = (x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2.$$

Now the superficial forces necessary to maintain equilibrium, with u_1, v_1, w_1 for displacements, are L_1, M_1, N_1, where

$$L_1 = -2b^2\frac{dU}{dn} - (a^2 - 2b^2)\theta\frac{dx}{dn} - b^2\left(r\frac{dy}{dn} - q\frac{dz}{dn}\right),$$

U being written for u on the surface; or

$$L_1 = 2b^2(a^2 - b^2)\left\{\frac{d}{dn}\frac{d^2\rho}{d\xi^2} - 2\frac{d\xi}{dn}\frac{d}{d\xi}\frac{1}{\rho}\right\} - 2a^2b^2\frac{d}{dn}\frac{1}{\rho},$$

and M_1, N_1 have corresponding meanings.

And now, making use of Betti's reciprocity theorem, as in the treatise "Sulle equazioni della elastica," above abstracted, with $u_1\, v_1\, w_1$ for the auxiliary displacements, we have, when the mass forces X, Y, Z are not zero, and there are impressed forces L, M, N on the surface,

$$8\pi a^2 b^2 u = k\iiint (Xu_1 + Yv_1 + Zw_1)\, dx\, dy\, dz$$
$$+ \iint(LU_1 + MV_1 + NW_1)\, dS - \iint(L_1U + M_1V + N_1W)\, dS, \&c. \quad (4)$$

III. It is now required to find values of the generating functions ϕ, ψ, χ which will reduce equations (2) to the form (4). Taking the three terms in the right-hand member of (4) separately, it is found that the system

$$\phi' = \frac{1}{8\pi a^2 b^2}\int X\rho\, dx\, dy\, dz + \frac{1}{8\pi a^2 b^2}\iint L\rho\, dS,$$

will satisfy the condition for the first two terms. Again, each of the two systems

$$\phi_1 = \tfrac{1}{2}\frac{\rho}{dx}\frac{d\xi}{dn} - \tfrac{1}{2}\frac{d\rho}{dn},$$

$$\psi_1 = \tfrac{1}{2}\frac{d\rho}{dx}\frac{d\eta}{dn},$$

$$\chi_1 = \tfrac{1}{2}\frac{d\rho}{dx}\frac{d\zeta}{dn},$$

and

$$\phi_2 = \frac{a^2 - 2b^2}{2b^2} \frac{d\rho}{dx} \frac{d\xi}{dn},$$

$$\psi_2 = \frac{a^2 - 2b^2}{2b^2} \frac{d\rho}{dy} \frac{d\xi}{dn},$$

$$\chi_2 = \frac{a^2 - 2b^2}{2b^2} \frac{d\rho}{dz} \frac{d\xi}{dn},$$

satisfies (1) if $X = 0$ &c.; and therefore the system $\phi_1 + \phi_2$ &c. satisfies it. And if we derive u, v, w from $\phi_1 + \phi_2$ &c. as generating functions, we find

$$u = \frac{1}{2b^2} L_1, \quad v = \frac{1}{2b^2} M_1, \quad w = \frac{1}{2b^2} N_1.$$

It follows that if we make

$$\phi'' = -\frac{1}{4\pi a^2} \cdot \iint (\phi_1 + \phi_2) U \, dS,$$

&c.,

and we derive $u_1 \, v_1 \, w_1$ from $\phi'' \, \psi'' \, \chi''$ as generating functions, we get

$$u_1 = -\frac{1}{8\pi a^2 b^2} \cdot \int L_1 U \, dS,$$

$$v_1 = -\frac{1}{8\pi a^2 b^2} \cdot \int M_1 U \, dS,$$

$$w_1 = -\frac{1}{8\pi a^2 b^2} \cdot \int N_1 U \, dS.$$

In the same way we can find the values of u, v, w when V, W are put for U in the right-hand member of these equations; and so we have found the generating functions from which can be derived U, V, W in (4).

IV. The problem of determining $\phi \, \psi \, \chi$ when $u \, v \, w$ are known is that of integrating equation (2). For this purpose assume

$$\phi = (a^2 - b^2) \frac{d}{dx} \left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} \right) + b^2 \Delta_2 F,$$

$$\psi = (a^2 - b^2) \frac{d}{dy} \left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} \right) + b^2 \Delta_2 G,$$

$$\chi = (a^2 - b^2) \frac{d}{dz} \left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} \right) + b^2 \Delta_2 H.$$

Then we find

$$\Theta = a^2 \Delta_2 \left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} \right),$$

$$P = b^2 \Delta_2 \left(\frac{dH}{dy} - \frac{dG}{dz} \right), \quad Q = b^2 \Delta_2 \left(\frac{dF}{dz} - \frac{dH}{dx} \right),$$

$$R = b^2 \Delta_2 \left(\frac{dG}{dx} - \frac{dF}{dy} \right);$$

and equations (2) become

$$a^2 b^2 \Delta_2 \Delta_2 \mathrm{F} = u,$$
$$a^2 b^2 \Delta_2 \Delta_2 \mathrm{G} = v,$$
$$a^2 b^2 \Delta_2 \Delta_2 \mathrm{H'} = w \, ;$$

and integrals of these are

$$\mathrm{F} = -\frac{1}{8\pi a^2 b^2} \iiint u\rho \, dx \, dy \, dz + \iint f\rho \, d\mathrm{S} + \iint f' \frac{1}{r} \, d\mathrm{S},$$
$$\mathrm{G} = -\&\mathrm{c.},$$

in which f, f' are arbitrary functions of the coordinates of a point on the surface S.

V. The author now considers two particular integrals of the equation

$$\Delta_2 \Delta_2 \omega = 0,$$

namely,

$$\omega = z\Omega, \qquad \omega = (\mathrm{R}^2 - \rho^2)\Omega,$$

in which $\rho = \sqrt{x^2 + y^2 + z^2}$, R is a constant, and Ω satisfies Laplace's equation, $\Delta_2 \Omega = 0$. The first corresponds to a solid bounded by a plane $z = 0$, the second to a sphere $\rho = \mathrm{R}$. If we take for generating functions, functions which have these forms, we obtain those integrals of the equations of equilibrium by which the problem has been solved of finding the deformation in a solid bounded by a plane, or, as the case may be, a sphere. For instance, in case of the plane assume

$$\phi = z\Phi, \qquad \psi = z\Psi, \qquad \chi = z\mathrm{X},$$
$$\Sigma = \frac{d\Phi}{dx} + \frac{d\Psi}{dy} + \frac{d\mathrm{X}}{dz}.$$

Then

$$u = (b^2 - a^2) \left\{ z\frac{d\Sigma}{dx} + \frac{d\mathrm{X}}{dx} \right\} + 2a^2 \frac{d\Phi}{dz},$$
$$v = (b^2 - a^2) \left\{ z\frac{d\Sigma}{dy} + \frac{d\mathrm{X}}{dy} \right\} + 2a^2 \frac{d\Psi}{dz},$$
$$w = (b^2 - a^2) \left\{ z\frac{d\Sigma}{dz} + \frac{d\mathrm{X}}{dz} \right\} + 2a^2 \frac{d\mathrm{X}}{dz} + (b^2 - a^2)\Sigma,$$

But these can be divided into two systems :

$$u_1 = (b^2 - a^2) \left\{ \frac{d\mathrm{X}}{dx} - \frac{d\Phi}{dz} \right\},$$
$$v_1 = (b^2 - a^2) \left\{ \frac{d\mathrm{X}}{dy} - \frac{d\Psi}{dz} \right\},$$
$$w_1 = (b^2 - a^2) \left\{ \frac{d\Phi}{dx} + \frac{d\Psi}{dy} + \frac{d\mathrm{X}}{dz} \right\} ;$$

and

$$u_2 = (b^2-a^2)z\frac{d\Sigma}{dx}+(a^2+b^2)\frac{d\Phi}{dz},$$

$$v_2 = (b^2-a^2)z\frac{d\Sigma}{dy}+(a^2+b^2)\frac{d\Psi}{dz},$$

$$w_2 = (b^2-a^2)z\frac{d\Sigma}{dz}+(a^2+b^2)\frac{dX}{dz},$$

In these forms are included the integrals obtained by Boussinesq and Cerruti for the solution of the problem of the deformations in a solid bounded by a plane.

Then follows a somewhat similar treatment for the case of a sphere.   S. H. B.

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286. *General Formula for the representation of a Field of Force by Elastic Forces.* **C. Somigliana.** (R. Inst. Sc. Lett. Lombardy, 18 Dec., 1890.)—In this paper Signor Somigliana proposes the following problem :—To determine the deformation produced in an indefinitely extended homogeneous isotropic elastic medium by given forces X, Y, Z acting on a finite part of the mass, and by superficial forces L, M, N acting on certain finite surfaces, subject to the condition that the components of displacement undergo given discontinuity H_1, H_2, H_3 in crossing certain surfaces, and vanish at an infinite distance.

The field resembles Maxwell's field, and the displacements are subject to the same conditions as his so-called components of electric displacement f, g, h; but to begin with the given forces are of the most general character, and are subsequently restricted to forces according to the Newtonian law derived from a potential V. The problem is the same as that of finding a deformation of the medium which will produce at every point forces equal and opposite to those of the given field. Using the method developed in the paper " Sulle equazioni della elasticita," above abstracted, we have an auxiliary system of displacements,

$$u' = \frac{B-A}{A}\frac{d^2\frac{r}{2}}{d\xi^2} + \frac{1}{r},$$

$$v' = \frac{B-A}{A}\frac{d^2\frac{r}{2}}{d\xi\,d\eta},$$

$$w' = \frac{B-A}{A}\frac{d^2\frac{r}{2}}{d\xi\,d\zeta},$$

for the components of a deformation which would be in equilibrium under the action of certain determinate superficial forces L', M', N', and then the values of u, v, w required to produce the given field of force are at a point $x\,y\,z$ at a finite distance from the surface of discontinuity :

$$u\,(x\,y\,z) = \frac{1}{4\pi\mathrm{B}}\,\rho\iiint(\mathrm{X}u' + \mathrm{Y}v' + \mathrm{Z}w')\,dx\,dy\,dz$$
$$+ \frac{1}{4\pi\mathrm{B}}\iint(\mathrm{L}u' + \mathrm{M}v' + \mathrm{N}w')\,d\mathrm{S}$$
$$+ \frac{1}{4\pi\mathrm{B}}\iint(\mathrm{L'U} + \mathrm{M'V} + \mathrm{N'W})\,d\mathrm{S},$$

U, V, W being the values of u, v, w on the surface. It is shown that the surface-integrals vanish if extended over a sphere of radius R when R is infinite.

Let now u &c. have the discontinuity H_1 &c. in crossing certain closed surfaces σ_1, σ_2, &c., and certain unclosed surfaces τ_1, τ_2, &c. All space may be divided into two regions by extending the τ surfaces so as to form a single closed surface T, within which are the closed surfaces σ, the one part of space being self-connected throughout, and the other part consisting of (1) space outside of T, (2) space inside of σ. These two regions of space are separated by the surface S. Applying the formula to that region within which is situated the point $x\,y\,z$, we have the above equation. Applying it to the other region we have an equation of the same form with zero on the left-hand side instead of $u\,(x\,y\,z)$. The surface-integrals are over the same surface S in the two cases. And since on the surface we have the given discontinuity H_1, H_2, H_3 for the displacements u, v, w, the result becomes

$$u\,(x\,y\,z) = \frac{1}{4\pi\mathrm{B}}\iiint\rho\,(\mathrm{X}u' + \mathrm{Y}v' + \mathrm{Z}w')\,dx\,dy\,dz$$
$$+ \iint(\mathrm{K}_1 u' + \mathrm{K}_2 v' + \mathrm{K}_3 w')\,d\mathrm{S}$$
$$+ \iint(\mathrm{H}_1\mathrm{L'} + \mathrm{H}_2\mathrm{M'} + \mathrm{H}_3\mathrm{N'})\,d\mathrm{S},$$

K_1 being written for $\mathrm{L}_1 + \mathrm{L}_2$, or the sum of the x components of the given superficial forces acting on the two sides of S respectively. It is now shown that there cannot be two separate sets of values for u &c. satisfying the above conditions. The above holds if $\mathrm{H} = 0$, or the forces are continuous throughout all space.

Now, considering this last case, let us write x_i, in which i is 1, 2, or 3, for x, y, z respectively; and, similarly, u_i for u, v, or w, &c. Introduce the notation

$$\mathrm{F}_i = -\frac{1}{4\pi\mathrm{B}\rho}\iiint k\mathrm{X}_i\,\frac{1}{r}\,dx\,dy\,dz$$

$$f_i = -\frac{1}{4\pi\mathrm{B}\rho}\iint h\mathrm{X}_i\,\frac{1}{r}\,d\mathrm{S},$$

$$\mathrm{G} = \frac{\mathrm{B} - \mathrm{A}}{4\pi\mathrm{A}\mathrm{B}}\sum_{i=1}^{i=3}\iiint k\mathrm{X}_i\frac{d\,\frac{r}{2}}{d\xi_i}\,dx\,dy\,dz,$$

$$g = \frac{\mathrm{B} - \mathrm{A}}{4\pi\mathrm{A}\mathrm{B}}\sum_{i=1}^{i=3}\iint h\mathrm{X}_i\frac{d\,\frac{r}{2}}{d\xi_i}\,d\mathrm{S},$$

where k is volume density, h superficial density. Also let α_{il}, in which

$$i = 1, 2, 3 \text{ successively,}$$
$$l = 1, 2, 3 \text{ successively,}$$

denote the six components of deformation whose types are

$$\frac{du}{dx_i} \quad \text{and} \quad \frac{du_i}{dx_l} + \frac{du_l}{dx_i}.$$

Then we get

$$\alpha_{ii} = \frac{d^2(G+g)}{dx_i^2} + \frac{d(F_i+f_i)}{dx_i},$$

$$\alpha_{il} = 2\frac{d_2(G+g)}{dx_i\,dx_l} + \frac{d(F_i+f_i)}{dx_l} + \frac{d(F_l+f_l)}{dx_i}.$$

And if T_{ii}, T_{il} denote the corresponding components of elastic tension,

$$T_{ii} = \frac{A-2B}{B-A} \cdot B\Delta_2(G+g) - 2B\left(\frac{d^2(G+g)}{dx_i^2} + \frac{d^2(F+f)}{dx_i}\right),$$

$$T_{il} = B\left(2\frac{d^2(G+g)}{dx_i\,dx_l} + \frac{d(F_i+f_i)}{dx_l} + \frac{d(F_l+f_l)}{dx_i}\right).$$

Further, if the forces are according to the Newtonian law with potential V, we must write in the formulæ above given for F, G, &c.,

$$X = \frac{dV}{dx}, \qquad k = -\frac{1}{4\pi}\Delta_2 V,$$

$$h = -\frac{1}{4\pi}\left(\frac{dU}{d\nu} + \frac{dV}{d\nu'}\right),$$

in which $d\nu$, $d\nu'$ are elements of the normal to, and on opposite sides of, the separating surface S. And the equations so formed are the equations which will take the place of Maxwell's field equations.

Finally, our author investigates the modification required in the above formulæ when the point $x\,y\,z$ is on one of the surfaces of discontinuity.　　　　　　　　　　　　　　　　　　　　　　S. H. B.

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287. *Specific Heat of Tungsten.* **A. W. Grodspeed** and **E. F. Smith.** (Zschr. an. Chem. 8. pp. 207–212, 1895.)—The method employed to determine the specific heat is exactly the same, in principle, as that described by Joly in Proc. Roy. Soc. (1886) 41. pp. 248, 352, under the title "On the Method of Condensation in Calorimetry." Joly's brass calorimeter is replaced by a glass sphere of about 1 dm. diameter, which has a neck 3 cm. diam. extending upwards for about 1 dm. The top of the neck is covered with a circular piece of thin specular stone containing a central opening of 2 mm. diam. The sphere has three additional openings—one for the inlet of steam, another for the outlet, and

the third for the introduction of a thermometer.   The substance,
whose Sp. H. is to be determined from observation of the mass of
steam condensed, &c., hangs inside the sphere by means of a
platinum wire, the upper end of which is attached indirectly to the
extremity of a beam of a balance.   As the tungsten was in the
form of a fine powder, it was placed in a thin glass bulb of known
mass and caloric capacity.   The authors find that the Sp. H. of
Tungsten is 0·0338 with a probable error of $\pm 0·0003$; if its
atomic weight be taken as that obtained by Smith and Pennington,
viz. 184·921, its atomic heat is 6·25 ; if that obtained by Smith
and Desi, viz. 184·704, its atomic heat is 6·243.   A. G.

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288. *Effect of Heat and Electric Current on Young's Modulus.*
M. C. Noyes. (Phys. Rev. 2. pp. 277–297, 1895.)—The ex-
periments described were undertaken in order to see whether the
elasticity of steel reaches a maximum somewhere between 100° C.
and 200° C., as Wertheim's results indicate, or whether it decreases
continuously with increase of temperature, as other observers have
found.   A steel pianoforte-wire was stretched horizontally on a
comparator and fixed at one end ; its other end was passed over a
friction-wheel and attached to a weight by which it was stretched.
Two marks on the wire about 890 m. apart were viewed by the
microscopes of the comparator.   The wire was sometimes heated
by a solenoid of german-silver wire surrounding it and traversed
by a current : in this case the temperature was determined by
thermometers inside the solenoid and near the wire.   Sometimes,
however, the current was passed through the test-wire itself, and
the temperature was then measured by the expansion or the in-
crease of electrical resistance of the wire.   Before experiment
each wire was stretched for 24 hours or more by a weight greater
than the maximum one used during the observations.   The results
show that the elasticity decreases with rise of temperature, the rate
of decrease being about 0·05 per cent. per degree Centigrade ; this
value agrees with the observations of Kohlrausch and Loomis.
The maximum found by Wertheim is therefore apparently spurious.
A wire heated to about 200° C., and then cooled again has its
elasticity increased, probably on account of annealing.   An electric
current through the wire alters its elasticity otherwise than by
merely heating it.   Small currents increase the modulus in spite
of the rise of temperature which they produce, but with stronger
currents the heating effect seems to predominate.   Elasticity is
not appreciably affected by the longitudinal magnetisation of the
wire produced by the solenoid surrounding it.   J. L. H.

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289. *Should the Astronomical coincide with the Civil Day ?* **H.
Poincaré.** (Ann. Bur. Long. 1895.)—In Oct. 1894 the French
Minister of Public Instruction asked the Bureau des Longitudes
to give its opinion of a proposition made by the Canadian Institute
and the Astronomical Society of Toronto, that the astronomical
day, like the civil, should begin at midnight.   The paper under

consideration is a *Report* in reply. As early as 1804, owing to Laplace's influence, the Bureau passed a resolution in favour of the change: it was not, however, carried out, although Laplace himself adopted it, and many compilers of tables followed his example until the time of Leverrier. An International Conference held at Washington in 1884 adopted a resolution in favour of the change; in 1885 an Astronomical Congress held at Geneva discussed the resolution ; the majority criticised it, in particular the directors of the ' Berliner Jahrbuch.' In 1885 a half-measure was adopted at Greenwich : spectroscopic, photographic, and magnetic observations are recorded by means of a day beginning at midnight, the hours being counted from 0 to 24. So matters stood when the Canadian Institute and the Astronomical Society of Toronto appointed a Commission to examine the question. The Commission, evidently favourable to the reform, sent out a circular to all known astronomers asking for their opinion: result—108 in favour of change, 63 against. The majority of the German replies were antagonistic, but the majority of the Russian, Austrian, English, American, Italian, and French were favourable. The Lords of the Admiralty think that the change to be useful must be universal. The author gives as reasons against the change, (1) midnight is the middle of an astronomer's working day, and it would be inconvenient for him to change the date at the period of greatest activity ; (2) the discontinuity produced in the record of time would be a trouble. *In favour*, (1) the mariner, an astronomical observer for the time being, makes his observations of the sun at mid-day ; (2) uniformity is intrinsically desirable ; (3) the change will doubtless be made in the future—better sooner than later. Finally, the Bureau's resolution is given, which implies that it is favourable in principle to the change : that it thinks that, for the reform to be effectual, an understanding must exist between the Governments publishing the principal tables : and that uniformity will only be complete when in the civil day, as in Italy, the hours are counted successively from midnight to midnight. A. G.

290. *Saturation-Pressure of a Vapour in an Electric Field.* **A. Sokolow.** (J. de Phys. 4. pp. 53–62, 1895.)—The state of a charged condenser, consisting of two large metal plates separated by thin layers of an insulating liquid and its vapour, is assumed to be perfectly determined by the temperature t, the volume of the dielectric v, and its electric charges $\pm E$; in this case the pressure p would be a function of t and E. Then if t is kept constant, an increase of charge dE will entail an increase of volume dv, so that if the alteration of volume occurs without alteration of the distance between the plates (*e. g.* by the motion of a piston in a small horizontal tube leading from that part of the non-conducting boundary of the fluid dielectric which is in contact with vapour), the work required for this operation is $x\,dE - p\,dv$, where x is the potential-difference of the plates, and as the temperature remains constant this is the differential of the free energy, and is thus a perfect

differential, or $\partial x/\partial v = -\partial p/\partial \mathrm{E}$: but $x = \partial \mathrm{W}/\partial \mathrm{E}$, if $\mathbf{W}$ is the energy of the condenser, so that

$$\frac{\partial}{\partial \mathrm{E}}\left(p + \frac{\partial \mathrm{W}}{\partial v}\right) = 0, \quad \text{or} \quad p = p_0 - \frac{\partial \mathrm{W}}{\partial v},$$

p_0 being the pressure with the condenser uncharged. If the volumes of liquid and vapour between the plates are v_1 and v_2, their specific volumes being σ and s, we obviously have

$$- dv_1/dv = dv_2/dv = \sigma/(s-\sigma) ;$$

and if we assume that the electric field causes an inward pressure q on the liquid we shall have $q d_1 = (\partial \mathrm{W}/\partial v)dv$, which gives

$$p = p_0 + q\sigma/(s-\sigma), \quad q = \partial \mathrm{W}/\partial v_1 - \partial \mathrm{W}/\partial v_2.$$

With horizontal plates of area $\mathbf{S}$, $\mathbf{W} = 2\pi \mathrm{E}^2 \mathrm{S}^{-2}(v_1/k_1 + v_2/k_2)$, where k_1, k_2 are the specific inductive capacities of the liquid and vapour, and therefore $q = 2\pi \mathrm{E}^2 \mathrm{S}^{-2}(1/k_1 - 1/k_2)$; and with vertical plates separated by δ, $\mathrm{W} = 2\pi \mathrm{E}^2 \delta^2/(k_1 v_1 + k_2 v_2)$, whence

$$q = 2\pi \mathrm{E}^2 \delta^2 (k_2 - k_1)/(k_1 v_1 + k_2 v_2)^2.$$

[In the original there occurs some confusion of signs.]

These same expressions are also obtained by consideration of a cycle in which alterations of charge and volume occur isothermally; but two mistakes occur (which, however, neutralise each other), viz., $(\partial \mathrm{W}/\partial v_1)\delta v$ for $(\partial \mathrm{W}/\partial v_1 - \partial \mathrm{W}/\partial v_2)\delta v$ as the equivalent for $\mathrm{W}_1' - \mathrm{W}$, and $\partial \mathrm{W}/\partial v_1 . \sigma/(s-\sigma)$ for $(\partial \mathrm{W}/\partial v_1 - \partial \mathrm{W}/\partial v_2)\sigma/(s-\sigma)$ as the equivalent of $-\partial \mathrm{W}/\partial v$.　　　　　R. E. B.

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291. *The Animal as a Prime Mover.* **R. H. Thurston.** (J. Frank. Inst. no. 829. pp. 1–20, Jan. 1895.)—In this paper an attempt is made to evaluate the power supplied to the human mechanism considered as a prime mover ; and to assign numerical values to the losses in the various functions of the animal machine. The expenditures, thermal, mechanical, and mental, are as nearly as possible summed up ; and the result is compared with the total energy supplied in the form of food. The question of the relative nutritive energies of different foods is carefully considered ; the facts being presented chiefly in the form of data selected from acknowledged authorities, together with some more recent results of experiments upon inmates of the penal institutions of New York. A study is made of a tabulated summary of the nutritive and heat-producing elements in the best dietaries ; from which it is deduced that the power represented by all the active phenomena, internal as well as external, of the human machine, supposed to be fully working for a whole day, would be equivalent to that supplied by a pound of coal burning uniformly in the same time. In this table the nutrients are classed as including the muscle of meats, the casein of milk, and the gluten of grains ; while the fats and carbohydrates are regarded as purely heat-producing ; a diagram sets out the dietaries of various phases of humanity in a very useful form. The power of the animal machine is shown to be very great when emergency calls for special effort. Thus, a man

who develops only one-eighth of a horse-power for an average day's work, can exert a half horse-power for two or three minutes; and a full horse-power for a few seconds. The aggregate power of the machine varies from an indefinitely small quantity with the minuter forms of life, and attains, in the case of a whale swimming at the rate of 40 knots an hour, to 140 horse-power. Assuming that only external work is reckoned, and that the latent energy of the food-supply is unity, the efficiency of the animal machine is computed at about 20 per cent. The estimates for the efficiency of the mental mechanism are more difficult to make, and the figures are probably not of much worth. R. A.

292. *Atomic Weight of Argon.* **Lecoq de Boisbaudran.** (C. R. 120. pp. 361, 362, 1895.)—The author believes argon to belong to a group of elements having atomic weights $20 \cdot 0945$; $36 \cdot 40 \pm 0 \cdot 08$; $84 \cdot 01 \pm 0 \cdot 2$; $132 \cdot 71 \pm 0 \cdot 15$ ($0 = 16$). The atomic value of this series should be even (octads) and the two elements with lowest atomic weights abundant in nature, whilst the elements with atomic weight $84 \cdot 01$ and $132 \cdot 71$ will be rare. The first of the series should be more volatile than oxygen and the second than sulphur, and the other two elements should be similarly related to selenium and tellurium. S. R.

293. *Effect of Pressure on the Temperature of Change of Allotropic Form.* **S. Lussana.** (N. Cim. 4. i. pp. 97–108.)—Clapeyron established the formula $l = A \, dp/dt$, where l is the latent heat, due to a change of unit volume, of a body at constant temperature, A is a constant depending only on the temperature, and p the pressure exerted on the body. Afterwards Kelvin showed that $A = T/E$, where T is the absolute temperature and E the mechanical equivalent of heat. Applied to the case of a body passing from the solid to the liquid state, the formula becomes

$$\Delta t = \frac{T(v' - v)}{EC} \Delta p,$$

where Δt is the change in the melting-point caused by a variation of pressure Δp; $v' - v$ is the variation of volume which occurs when the body passes from the solid to the liquid state, and C is the latent heat of fusion. The accuracy of the formula, with certain restrictions indicated by theory, has been confirmed experimentally. A phenomenon analogous to fusion is molecular transformation. After an interesting *résumé* of Reichert's work on the dependence on pressure of the temperature at which sulphur changes its allotropic form, and of Mallard and Le Chatelier's work on the dimorphism of silver iodide, the author describes the apparatus he employed to study the effect of pressure on the temperatures of transformation of ammonium nitrate and mercuric iodide. The substance experimented on is placed in a glass vessel enclosed in a strong iron tube, connected in a suitable manner with a Cailletet's pressure-pump. The temperature is measured by means of a thermoelectric couple consisting of a joint of

platinum and platinum-rhodium. The temperature at which the change takes place is measured in two ways: (1) by observing the rate at which the temperature of the body rises when it is steadily supplied with heat—at the point of transformation the temperature is either constant for some time or, as is more generally the case, falls temporarily ; (2) by observing the rate at which the temperature *falls* when the body is cooled. The results obtained are indicated in the following table :—

Name of salt.	p in atmospheres.	t rising.	t falling.	Δt rising.	Δt falling.	Δt calc.
Ammonium nitrate: 1st transformation.	1	35·45	30·55			
	50	37·05	31·98	+1·6	+1·43	+1·67
	100	38·59	33·37	3·14	2·82	2·94
	150	39·77	35·20	4·32	4·65	4·41
	200	41·47	36·44	6·02	5·89	5·83
	250	42·76	37·95	7·31	7·40	7·35
Ammonium nitrate: 2nd transformation.	1	85·85				
	50	85·15	......	−0·70	......	−0·70
	100	84·38	......	1·47	......	1·40
	150	83·73	......	2·12	......	2·10
	200	83·03	......	2·82	......	2·80
	250	82·29	......	3·56	......	3·55
Ammonium nitrate: 3rd transformation.	1	125·60	124·90			
	50	126·18	124·55	+0·58	+0·65	
	100	126·80	126·05	1·20	1·15	
	150	127·48	126·64	1·88	1·74	
	200	127·91	126·26	2·31	2·36	
	250	128·75	127·90	3·15	3·00	
Mercuric iodide.	1	137·24				
	50	137·30	......	+0·06	......	+0·55
	100	137·39	......	0·15	......	1·10
	150	137·44	......	0·20	......	1·65
	200	137·52	......	0·28	......	2·20
	250	137·63	......	0·39	......	2·75

The first column gives the name of the salt ; the second the pressure ; the third the temperature of transformation obtained with rising temperature ; the fourth the same obtained with falling temperature ; the fifth and sixth the variations caused by pressure ; the seventh the variations calculated from Kelvin's formula. The values of Δt, in the case of ammonium nitrate, are calculated from observations made by Bellati and Romanese ; in the case of mercuric iodide, Rodwell's determination of the change of density with pressure and Schwartz's determination of the heat of fusion are used—the author doubts the accuracy of the data. The author concludes that Kelvin's formula is confirmed quantitatively in the case of the first and second transformations of ammonium nitrate and qualitatively in the case of mercuric iodide. Assuming the truth of the formula, the variation of volume in the third transformation of ammonium nitrate is 0·01465.   A. Gs.

LIGHT.

294. *Optical Thin-Plate in the Case of Total Reflection.* **C. Fabry.** (C. R. 120. pp. 314–317, 1895.)—If the hypothenuse-faces of two prisms, one such face being slightly convex, are placed together and viewed in the direction corresponding to total reflec-tion, the apparatus presents a transparent spot, which is not confined to the point of contact but extends to parts separated by a thickness of air equal to several wave-lengths. The spot is dark if viewed by reflection. This phenomenon, noticed by Newton and Fresnel and completely studied by Quincke, is here accounted for mathematically. Let the incident ray be polarised in the plane of incidence, and let h be the reflection-coefficient for a single reflection at the thin-plate of thickness e. Then, taking account of the multiple reflections, I, the intensity of the transmitted ray may be expressed as a function of h and e. Now, when the critical angle is exceeded, h becomes imaginary but I is still real. After several transformations it is shown that, in the case of total reflection, the expression for I no longer involves a *sine* function of e but involves instead an *hyperbolic-sine* function. This agrees with the absence from the experiment in question of the periodic maxima and minima so conspicuous in the ordinary case of thin-plate inter-ference. Expressions are also given for the transmitted and reflected rays produced by an incident ray (1) polarised in the second principal azimuth, and (2) polarised in any other azimuth. In the last case the resulting rays are elliptically polarised. It is shown in detail that the mathematical expressions obtained com-pletely account for the experimental phenomena observed by Quincke. E. H. B.

295. *Photo-Theodolite.* **O. Ney.** (Zschr. Instr. 15. pp. 55–58, Feb. 1895.)—This is an improved form of a photo-theodolite—an instrument designed as an auxiliary to the ordinary theodolite, for facilitating the preparation of topographical charts and archi-tectural plans. It may be regarded, in its present form, as a photographic camera, with theodolite adjustments, provided with a lens giving accurate perspective delineation. The position of an object is determined by measurements derived from photographs taken at two or more points of observation—apparently by treating these as perspective drawings and plotting out the distance of the given object behind the picture-plane. The method is not described, the author reserving his account of the mechanical device which gives the requisite localising data from the photographs for a future communication. R. A.

296. *Photographic Photometry.* **J. Janssen.** (Ann. Bur. Long. 1895.)—This is a brief summary of the author's work

during the past thirteen years on the subject of astronomical photometry. The method employed consists in comparing the times required by the two sources of light to produce impressions on photographic films of equal sensitiveness which, when developed, give negatives of the same density. For stars, the image if focussed would be a mere point, so the film is placed slightly nearer the object-glass of the telescope used than the focus ; the impression on the film is then a circle of practically uniform illumination, called by the author a *stellar circle*. A star of the first magnitude produces a measurable effect in a few seconds, so that several observations with different times of exposure can be made on the same plate and developed at the same time. Having exposed and developed a plate for each of two stars to be compared, the circles which have the same density are selected, and the times of exposure corresponding to them are inversely as the intensities of the light from the two stars. In order to compare the sun with a star a screen is used, pierced with holes of the same diameter as the stellar circles. This is placed against the surface of the film and behind a shutter containing a triangular aperture. The shutter moves rapidly across the field of view, and exposes the various holes to the sun's light for different times which can be measured. The circle which on development coincides in density with one of the star-circles is selected, and comparisons of times of exposure are made as above.

The principle of graduating exposure by shape of aperture has been extended by the author in order to measure the intensity of light at different points of a comet's tail. A vertical slit, shaped like the tail of a comet, is uniformly illuminated, and a photograph of it is taken by means of a shutter moving horizontally ; this shutter has an aperture bounded by a rectilinear base and curved sides, the curves being of the form $y = ax^m$, with the axis of y horizontal. Shutters were made with m equal to 1, 2, 3, 4, ... The photograph obtained will represent a comet with the intensity of light at any point of its tail varying as the inverse mth power of the distance of the point from the nucleus of the comet. These photographs are compared with that of the actual comet, and the law of variation of intensity is found. For the comet b 1881 the law of intensity was found to be between the inverse fourth and inverse sixth powers of the distance from the nucleus.

The author suggests a method of permanently recording the conditions under which photographs of nebulæ are taken. This consists in simultaneously taking photographs, on the same plate, of the stellar circles of one or two well-known invariable stars in its neighbourhood. Changes in the brightness of nebulæ can thus be detected, if such exist.   J. L. H.

297. *Measurement of Intensity of Light by Chemical Action ; Experiments with Mixtures of Ferric Chloride and Oxalic Acid.* **G. Lemoine.** (C. R. 120. pp. 441–444, 1895.)—Mixtures of ferric chloride and oxalic acid are decomposed in the presence of

light to an extent depending on the intensity of the light. The author argues that this cannot be, as some have supposed, an action started by the light and then continued independently, because the action ceases on the removal of the light. Also it cannot be due to heat, because the temperature at which decomposition takes place is much lower than when the mixture is heated in the dark. He considers that the vibratory movement of the ether communicated to the molecules of the liquid breaks down certain bonds which prevented the molecules from combining among themselves, and that these bonds must be continually broken down if the action is to be permanent. The process requires energy, which is taken from the light and is proportional to the intensity of the illumination. A method of allowing for the physical absorption of light by the liquid is indicated, and some results are given which were obtained from sunlight under different conditions. By the use of coloured screens or liquids the intensities of the various constituents of sunlight at different times can be compared. Their ratio varies with the seasons, the blue rays being most intense in the summer, while in winter the light is relatively richer in yellow rays.   **J. L. H.**

298. *Movements of the Planets deduced from the Displacement of the Bands in their Spectra.* **H. Deslandres.** (C. R. 120. pp. 417–420, 1895.)—The author has taken advantage of the favourable position of Jupiter to measure the displacement of the bands in the spectrum caused by the rotation of the planet. Photographs are taken of the spectra given by the light proceeding from either end of an equatorial diameter, together with a reference spectrum. If v is the linear velocity at the equator of the planet, the difference of linear velocity between points on opposite extremities of an equatorial diameter is $2v$; and since this difference of velocity affects both the incident light and the reflected light, the effective difference in velocity is $4v$. The values obtained for the quantity $4v$ vary between 46·9 and 50·2 kilometres per second, the value calculated from the accepted dimensions of Jupiter being 49·5 kilometres per second. The author points out that, although the radial velocity of the moon with reference to the earth is small, it is not small with respect to the sun, since it varies between + and −1·5 kilometres per second. In the case of the planets the radial velocity with reference to the *sun* varies from +9·81 kilometres per sec. in the case of Mercury to +0·24 kilometre per sec. in the case of Venus. A correction will therefore have to be applied even with the somewhat low-power stellar spectroscopes now in use. If it were possible to use the large grating spectroscopes employed for work on the sun, for stellar purposes it would be possible to obtain, from measurements of the displacement of the bands, the value of the solar parallax in the case of observations on Mercury correct to 1 part in 250.   **W. W.**

299. *Some Observations on the Previous Paper.* **H. Poincaré.** (C. R. 120. pp. 420–421, 1895.)—Suppose we have a planet at a distance R from the earth and R′ from the sun. Then the time taken by a wave of light to travel from the sun to the planet and then to the earth will be $\dfrac{R+R'}{V}$, where V is the velocity of the light. The succeeding wave will leave the sun at a time τ after the first (if $V\tau = \lambda$), and during this time the distances R, R′ will have become $R + \tau \dfrac{dR}{dt}$ and $R' + \tau \dfrac{dR'}{dt}$. Hence the second vibration will reach the earth in a time $\tau + \dfrac{\tau}{V}\left(\dfrac{dR}{dt} + \dfrac{dR'}{dt}\right)$ after the first, and this quantity will be the apparent periodic time of the light considered. The displacement of the absorption-bands will therefore be proportional to $\dfrac{dR}{dt} + \dfrac{dR'}{dt}$. The above theoretical views are supported by the experiments of Deslandres recorded in the previous abstract. Light which reaches us after reflection at the surface of a planet has suffered a triple absorption: (1) by the sun's atmosphere, (2) by the planet's atmosphere, (3) by the earth's atmosphere. The bands due to (1) will be displaced to an amount proportional to $\dfrac{dR}{dt} + \dfrac{dR'}{dt}$, those due to (2) to an amount proportional to $\dfrac{dR}{dt}$, while those due to (3) will not be displaced at all. —————— W. W.

300. *Pyrheliometric Observations during a Solar Eclipse.* **A. Bartoli, E. Stracciati,** and **G. Raffo.** (N. Cim. 4. i. pp. 76–86, 1895.)—The observations were made (by means of an instrument described in a former paper) at Catania on April 16th, 1893, the sky being cloudless and the pressure of aqueous vapour in the air almost constant throughout the day. The eclipse occurred late in the afternoon, and readings of the total solar radiation were taken at short intervals from the early morning until the eclipse was nearly over, the altitude of the sun being simultaneously measured. Using Laplace's formula, the authors calculate the thickness of air traversed by the sun's rays for each altitude, and plot a curve giving the total radiation in terms of this thickness. The curve from the morning's observations is identical with that from the afternoon's until the beginning of the eclipse, from which the authors conclude that the whole of the solar radiation comes from the sun's actual disc and none from its surrounding atmosphere. During the eclipse it was found that the radiation was not proportional to the extent of the sun's surface still uncovered, but varied in such a way as to point to the radiation from the parts near the limb being greater than that from the centre; but the authors think that this result needs confirmation. J. W. C.

301. *A Universal Focometer.* **M. Weiss.** (Bull. Soc. Fr. Phys. 56. 1895.)—To measure the focal length of any lens system, the author uses a telescope of 10 cm. focal length, exactly adjusted for parallel rays and carrying a micrometer eyepiece divided to $\frac{1}{100}$ mm. If this be set up at some distance from a graduated scale, and the lens to be measured put between, the lens can be moved till an image of the scale is formed in the field of the eyepiece. When that is the case the scale is in the principal focus of the lens, and the relation holds—

$$F/C = f/c,$$

where F=focal length required, f=focal length of the telescope =10 cm., C=diameter of object, c=diameter of image, only the one adjustment being required. The object is a scale in cm., mm., or fractions, according to the lens to be measured; if that be a microscope objective, the telescope may be let into the microscope-tube in place of its eyepiece; for larger lenses it is used on an optical bench. The telescope objective is made of two lenses of a little less than 20 cm. focal length each, screwed apart until of exactly the power required. R. A. L.

302. *Absorption of Light by Crystals.* **M. Camichel.** (Bull. Soc. Fr. Phys. 57. 1895.)—The apparatus used to measure the absorption is a modification of the spectrophotometer described by M. Gouy in the 'Annales de Chimie et de Physique,' 5. xviii. 1879. There are two collimators: before one is placed a pair of crossed Nicol's prisms, with a Soliel's compensator between; the crystal is placed in a suitable manner before the other collimator. The coefficient of absorption is calculated from observations made with the compensator. Conclusions :—(I.) The equations of vibration of the ether in the media studied—tourmaline, epidote, ferricyanide of potassium, andalusite—are linear. (II.) A single exponential suffices to represent the absorption of a vibration oblique with respect to the axes of optical elasticity; and the theory of the ellipsoid of absorption represents the phenomena in symmetrical crystals well. (III.) For unsymmetrical crystals the theory of the ellipsoid of absorption, as ordinarily employed, is not suitable; it is necessary at least to admit that the axes of the ellipsoid of absorption do not coincide with those of optical elasticity. With this restriction, the theory of the ellipsoid of absorption represents the phenomena well. (IV.) The obliquity of the axes of maximum and minimum absorption with the principal axes of optical elasticity (for vibrations in the plane of symmetry in the case of monoclinic crystals, and for vibrations in any principal plane whatever in the case of triclinic crystals) is a general fact in natural crystals and in crystals with intrinsic coloration. The crystals experimented on are epidote, the double sulphate of copper and cobalt, and axinite. In unsymmetrical crystals coloured artificially (De Senarmont's salt) the maxima and minima of absorption coincide, on the contrary, with the axes of optical elasticity. A. G

303. *Determination of Luminous Flux.* **A. Blondel.** (C. R. 120. pp. 311–314, 1895.)—The author gives a method of determining practically luminous flux : the method depends on the properties of diffusion by transmission. If α is the angle of incidence of ray of light on a thin diffusing screen, *e. g.* a Bunsen screen, and β the angle between a transmitted ray and the normal, the intensity I_β, of an element dS, of the surface of emission may be written

$$I_\beta = E f(\alpha, \beta)\, dS, \quad \dots \dots \dots \quad (1)$$

where E is the illumination of the surface of the incident rays, and $f(\alpha, \beta)$ is some function depending on α, β, and the material of which the screen is composed. For certain substances f is independent of α, and (1) may be written

$$I_\beta = E f(\beta)\, dS.$$

Let ρ be the distance between dS and the photometer screen,

l the perpendicular distance between S and the photometer-screen,

E' the illumination of the photometer-screen ;

then

$$E' = \int \frac{E f(\beta)\, dS}{\rho^2} = \int \frac{E f(\beta) \cos^2 \beta\, dS}{l^2} = \frac{K}{l^2} \int E\, dS$$
$$= \frac{K\phi}{l^2},$$

where ϕ is the flux to measured, and K is the mean value of $f(\beta) \cos^2 \beta$.

E' is balanced against some standard I' at a distance l' ; so that

$$\frac{K\phi}{l^2} = \frac{I'}{l'^2} \quad \text{and} \quad \phi = \frac{1}{K} \frac{l^2}{l'^2} \cdot I'. \qquad \text{W. G. R.}$$

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304. *Units of Light and Radiation.* **A. Macfarlane.** (T. Am. Inst. El. En. 12. pp. 85–98, 1895.)—Attention is drawn to the necessity of having a logical system of photometric units. M. Blondel's suggestions, which first appeared in ' La Lumière Electrique,' July 7, 1894, are discussed and extended. The author appears to advocate the universal adoption of the following units :—

Units.	Name.
The solid angle subtended by unit area on unit sphere ...	Steradian.
Luminous intensity of a " bougie," or one-twentieth part of the luminous intensity of the Violle standard ...	Pyr.
Luminous flux of a bougie through a steradian	Lumen.
Illumination produced by a flux of one lumen falling normally on unit area	Lux.
Quantity of illumination, corresponding to "exposure" in photography	Phot=Lux-second.
Quantity of light, or lumen-second	Rad.

The author also criticises the mathematical notations $\tan^{-1}$, $\sin^{-1}$, &c., and suggests, on the " mho " principle, that, if $y = \tan x$, we should write $x = \operatorname{nat} y$, &c. $\qquad$ W. G. R.

305. *Ultra-Violet Absorption*. **E. Nichols.** (Phys. Rev. 2. pp. 302–305, 1895.)—Two spectra arising from different portions of the same slit are simultaneously photographed side by side. The spectrum from the upper half of the slit is the record of unchanged sunlight; that from the lower half, of sunlight changed by passing through a layer of the substance under examination. The apparatus employed consists essentially of a Rowland concave grating of 2 m. radius and 568 rulings per mm., a narrow slit 4 cm. long, a heliostat with a silver mirror, and a camera. The photographs are taken upon a flexible celluloid film 20 cm. by 5 cm., exposed to the extreme violet and ultra-violet rays. Results are given for thirty-eight different substances. Of these, twelve show selective absorption in the region studied. Among them are several kinds of glass, a sample of white mica, solutions of uranium nitrate and uranium acetate, and a fluorescent solution. Reproductions of five of the more important photographs accompany the paper.

E. H. B.

306. *Photography of Manometric Flames*. **W. Hallock.** (Phys. Rev. 2. pp. 305–307, 1895.)—While researching upon the use of the nasal cavity in singing, it became desirable to record the manometric flames obtained. This led to the adoption of the following photographic method. A set of König resonators is used, tuned to a fundamental note and its first seven overtones, the images of the corresponding manometric flames being viewed in a rotating mirror. The camera is mounted upon a vertical axis passing approximately through the optical centre of the lens, and is rapidly turned by hand about this axis when taking a photograph. A sufficiently short exposure is further insured by placing between the mirror and the camera a black screen, having only a narrow slit to admit the light to the lens.

E. H. B.

307. *Distribution of Energy in the Spectrum of the Glow-Lamp*. **E. L. Nichols.** (Phys. Rev. 2. pp. 260–276, 1895.)—Two filaments are tested: one is bright, and the other smoked. The bulbs containing the filaments are joined together by a glass tube, so that the degree of vacuum may be the same in the two cases. The watts per candle-power required for the smoked filament are found to be much greater than for the bright one. The total energy in different parts of the spectrum is determined for each filament by means of a spectroscope and a delicate thermopile. The distribution of energy is found to differ in the two filaments, the maximum energy in the case of the bright filament being nearer the visible spectrum than in the case of the smoked filament. The paper is only an account of preliminary experiments.

W. G. R.

HEAT.

308. *Graphical Thermodynamics.* **René de Saussure.** (Am. J. Sc. 49. pp. 21–47, Jan. 1895: from Arch. Sci. phys. et nat. 31 May, 1894.)—M. de Saussure seeks in this paper to improve upon Clapeyron's well-known graphic representation of the states of a gas or other substance depending on pressure, volume, and temperature. For a given substance, any two of these three quantities being known, the third is determinate ; so the state of the substance may be graphically represented by the position of a point on a plane referred to two rectangular axes. In Clapeyron's system the abscissa v represents volume, the ordinate p represents pressure. M. de Saussure uses, instead of v and p, the following variables—namely, for abscissa $s = \pi a^2$, for ordinate $\phi = \dfrac{\pi}{i^2}$. Here a is the amplitude, i the periodic time, of the vibration of a particle of the substance under a force f varying as the displacement. We are asked to make the following fundamental hypothesis, viz. that heat consists in vibrations of the particles of the substance, and that the periodic time i and the amplitude a of such vibrations, and therefore also the mean velocity u, have at any instant the same values for all particles of the substance. We may consider the vibrations as the projections of a uniform circular motion with radius a under the central force f. Then $f = 2\pi^2 \dfrac{ma}{i^2}$. The abscissa s is called the *symbolical volume,* the ordinate ϕ the *symbolical pressure.* The whole heat at any instant in the substance is $\frac{1}{2}\Sigma m u^2$, and $\frac{1}{2}\Sigma m u^2 = \mathrm{KTE}$, in which E is the Joule equivalent, T the absolute temperature, and K is the *absolute specific heat.* The work done, internal and external, in any infinitesimal change in the state of the substance is shown to be equal to ϕds.

Comparing now M. de Saussure's diagram with Clapeyron's, we find that :—

(1) Whereas in Clapeyron's $p\,dv$ represents external work only, in M. de Saussure's the corresponding expression ϕds represents both the internal and the external work done.

(2) The whole heat in the substance, or $\Sigma \frac{1}{2}m u^2$, is proved to be equal to ϕs. Hence the isothermal lines in the diagram are the system of equilateral hyperbolas $\phi s = $ constant. The corresponding curves in Clapeyron's system, or $pv = $ constant, are isothermal lines only for so-called perfect gases.

(3) If $d\mathrm{H}$ denote a quantity of energy added, $d\mathrm{H}$ consists of two parts : (a) increase of heat, or $d(\phi s)$; (b) external and internal work done, or ϕds. That is,

$$d\mathrm{H} = d(\phi s) + \phi ds$$
$$= s\,d\phi + 2\phi\,ds.$$

Hence the adiabatic lines, or lines of constant H, are the system of curves $\phi s^2 = $ constant.
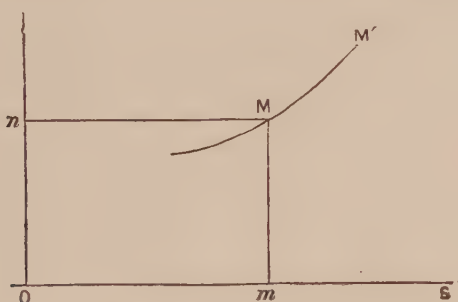
(4) Again, since

$$dH = \phi s \left(\frac{d\phi}{\phi} + 2\frac{ds}{s} \right)$$

we can prove Clausius' theorem that for a closed path

$$\int \frac{dH}{T} = 0.$$

This is known to be capable of proof by Clausius' method in all cases where the system possesses a definite periodic time i. It is a consequence of M. de Saussure's fundamental hypothesis.

On the diagram we may take a point M representing a state of the substance, and draw from M a line in any direction. If dH be the quantity of heat that has to be supplied to the substance when it passes from M to a neighbouring point on that line, dT the corresponding increase of temperature, $\frac{dH}{dt}$ is the specific heat under the conditions defined by the line, $e.\,g.$ at constant volume if the line be the direction in which the volume does not change. Let this, for any line, be denoted by γ. Now the absolute specific heat of the substance is denoted above by K, namely the ratio which the variation of the heat in the body ($\Sigma \frac{1}{2}mu^2$) bears to the variation of T. Then the values of γ for certain directions of the line are found, viz. :—

If the line be $s = $ constant, $\gamma = $ K ;

If the line be $\phi = $ constant, $\gamma = 2$K ;

If the line have a certain other defined direction, $\gamma = 3$K, which case is interesting because for certain substances the specific heat at constant volume is found to be three times the absolute specific heat. For such substances the last-mentioned line must be the line of constant volume. For perfect gases s and ϕ may be replaced by p and v. So for a perfect gas at constant volume $\gamma = $ K. This, which is a result of experiment, the author proves by mathematical reasoning as a consequence, namely, of his fundamental hypothesis. Certain physical properties of a substance can be

represented graphically ; for instance, the coefficients of dilatation and of compressibility.

The author also investigates the *Thermodynamic Function*, using that term indeed in a sense different from that in which it is employed by Rankine and Tait. In the works of the last-named writers the thermodynamic function is that function of volume and pressure which is constant along an adiabatic line. Our author treats $pv = Rt$, where R is constant, as the thermodynamic function for a perfect gas. Or, as he says, the thermodynamic function for any substance expresses the fact that the product of the symbolical volume by the symbolical pressure is proportional to the absolute temperature.   S. H. B.

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309. *Thermometer with a Platinum Bulb.*   **M. Marchis.** (Bull. Soc. Fr. Phys. 57.)—The cylindrical bulb, without soldering, one half millimetre in thickness, is attached directly to the stem of the thermometer. The filling presents some difficulties as mercury amalgamates with platinum above 150°. The following artifice is employed :—A vacuum is made in the apparatus, the bulb being raised to a red heat; it is allowed to cool, and then mercury is made to enter. The author shows by some experiments that there is a good vacuum, and that there is neither air nor amalgam present. The chief features of the thermometer are :—1t takes very quickly the temperature of its surroundings ; (2) its zero is "absolutely invariable"; experiments show that its zero is constant to nearly $\frac{1}{1000}$ of a degree.   A. G.

ELECTRICITY.

310. *Nature of Maxwell's Displacement Current.* **Vaschy.** (C. R. 120. pp. 255–258, 1895.)—In this communication M. Vaschy claims to explain the displacement-currents whose components in Maxwell's notation are $\dfrac{df}{dt}$ &c., without resorting to any hypothesis.

We may assume as an axiom that the energy within any volume bounded by the closed surface S cannot be increased or diminished otherwise than by the passage of energy through S. (Compare the theorem known as Poynting's.) The author requires us to assume further that the energy which does in fact pass through S in any short time dt depends only on the state of the electric field at the instant on S, and not in any degree on the state of things within S, whether, for instance, the medium within S be or be not conducting (A). The intensity of the electric field at any point is represented by a vector H, whose components are X, Y, Z. It is assumed that the energy per unit of volume is $\dfrac{\mathrm{H}^2}{4\pi k}$, where $\dfrac{1}{k}$ corresponds to Maxwell's K. Some of this energy is converted into heat, if there be any conductivity: namely, $\dfrac{\mathrm{H}^2}{\rho}$ per unit of time. Other possible transformations of it are not here considered.

This being the case, the energy which flows into an element of volume per unit of time is proportional to

$$\frac{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}{\rho}+\frac{1}{4\pi k}\left(\mathrm{X}\frac{d\mathrm{X}}{dt}+\mathrm{Y}\frac{d\mathrm{Y}}{dt}+\mathrm{Z}\frac{d\mathrm{Z}}{dt}\right),$$

that is, to $\qquad i_x\mathrm{X}+i_y\mathrm{Y}+i_z\mathrm{Z},$

if $i_x=\dfrac{\mathrm{X}}{\rho}+\dfrac{1}{4\pi k}\dfrac{d\mathrm{X}}{dt}$ and i_y, i_z have corresponding meanings. This may be taken as the definition of the vector whose components are i_x, i_y, i_z. But these are what Maxwell calls the components of the *total current*, in his notation $p+\dfrac{df}{dt}$, $q+\dfrac{dg}{dt}$, $r+\dfrac{dh}{dt}$.

It is then shown that

$$\frac{di_x}{dx}+\frac{di_y}{dy}+\frac{di_z}{dz}=0,$$

or the total current is closed. Having regard to the axiom (A), it is sufficient to prove this for the case in which the medium is a perfect insulator. In that case

$$\frac{di_x}{dx}+\frac{di_y}{dy}+\frac{di_z}{dz}=\frac{d}{dt}\left\{\frac{d}{dx}\left(\frac{\mathrm{X}}{4\pi k}\right)+\frac{d}{dy}\left(\frac{\mathrm{Y}}{4\pi k}\right)+\frac{d}{dz}\left(\frac{\mathrm{Z}}{4\pi k}\right)\right\}$$

$$=\frac{d\hat{o}}{dt},$$

O

if δ be the "*electric density*" at any point. But in an insulator, $\frac{d\delta}{dt}$ is always zero. This our author says is proved in Electrostatics. Therefore in an insulator

$$\frac{di_x}{dx} + \frac{di_y}{dy} + \frac{di_z}{dz} = 0.$$

And if in an insulator, then always by A.

From the above property of i_x, i_y, i_z may be derived a new vector, whose components X', Y', Z' satisfy

$$4\pi i_x = \frac{dY'}{dz} - \frac{dZ'}{dy} \quad \&c. ;$$

and, as shown by experiment, this new vector represents the intensity of the magnetic field, and it follows that the displacement current has formally the same magnetic properties as the conduction current. These properties of the displacement current, based by Maxwell on an hypothesis, the author claims to have established without hypothesis by strict mathematical reasoning. (*Cf.* Boltzmann, Vorlesungen, Part i. pp. 84, 104.)   S. H. B.

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311. *Resistance Determination in Absolute Measure.* **F. Himstedt.** (Wied. Ann. 54. pp. 305–334, 1895.)—For the purpose of determining the resistance of a conductor in absolute measure the author provides two circuits—a *primary*, formed of an induction-bobbin A, a resistance r_0, a rheostat K_0, an interrupter D_0, and a battery E; and a *secondary*, consisting of an induction-bobbin B, a rheostat K_1, an interrupter D_1, a galvanometer G, and a resistance $r = r_0$, to be measured. The bobbin A is a long solenoid concentric with the secondary coil B, which is slipped over it. Let the total resistance of the secondary circuit $= w + r$. Parallel to the resistance, r_0, in the primary circuit connect a resistance $w_0 = w$. When the interrupter, D_0, breaks the primary current, i, n times per second, and the interrupter, D_1, in the secondary circuit is so adjusted that only one set of the "make" and "break" impulses affects the galvanometer, there results a deflection a_1, and we have

$$G \cdot \tan a_1 = \frac{n \cdot i \cdot V}{r + w}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

where G is the galvanometer constant, and V the coefficient of mutual induction between A and B. The next operation is to disconnect the primary circuit at the ends of the parallel resistances r_0, w_0, cutting these resistances completely out; the two circuit wires thus freed are now joined, respectively, to the two ends of r in the secondary circuit, without changing the position of r, which remains in the secondary circuit. r and w are thus substituted in the primary circuit for r_0 and w_0. Both interrupters, D_1 and D_0, are now short-circuited and the galvanometer

is consequently deflected by a steady current through an angle a_2, in which case

$$G \tan a_2 = \frac{ri}{r+w}, \quad \cdots \quad \cdots \quad (2)$$

so that in order to determine r in absolute measure it is necessary to calculate V, and to observe n, and the angles a_1 and a_2. For the primary bobbin a solenoid was used, of great length compared to its radius, and also great in comparison to the length of the secondary bobbin. In this condition

$$V = 4\pi^2 R^2 . k . b (1-\gamma);$$

where R is the radius of the solenoid, k the number of turns per cm., b the number of windings on the secondary, and γ a correction coefficient, necessary because the solenoid is not infinitely long. The author gives an expression for γ in terms of the dimensions of the coils. In his actual experiments γ is, approximately, 3 per cent. of the total value of V. Lord Rayleigh had shown the necessity for exactness in winding the solenoid, and great care has here been taken to secure uniformity. The cylinder, upon which the solenoid is wound, is of glass ground accurately to a circular section. The wire is wound on in a dividing-engine, its tension being observed at a dynamometer, and kept constant. Further, the dividing-engine is geared to the paying-out drum, so that the amount of turning given to the glass cylinder is correctly proportioned to the distance through which it slides axially. Bare copper wire, specially non-magnetic, was used for the solenoid. The wire of the secondary was silk-covered and wound in a groove ground in a glass ring.

Measurements were made upon standard Hg resistances, of approximately 1 ohm each. The interrupter is designed for different speeds, and for picking out either the "make" or "break" impulses; contact is made between mercury, contained in concentric grooves cut in a horizontal fixed plate of vulcanite, and metallic brushes, carried by a revolving radial arm above the grooves, which dip vertically into the mercury. The circular grooves are subdivided by insulating spacing pieces. This apparatus with its speed-counting gear is said to have been completely successful. Check experiments were made (1) by using different solenoids; (2) by using different secondary coils, or different divisions of the same secondary coil; (3) by varying the rate and duration of interruption; (4) by utilizing the "make" or "break" impulses, respectively; (5) by varying the resistances added by k_0 and k_1 to the two circuits; (6) by using different standard resistances for r; (7) by using different galvanometers; (8) by varying the current. The results corresponding to these successive tests are tabulated, and the slight differences due to the interchanges can be conveniently traced. R. A.

312. *Determination of Induction-Coefficients of Solenoids.* **F. Himstedt.** (Wied. Ann. 54. pp. 335–341, 1895.)—The mea-

surement of the coefficients of self-induction of certain bobbins and galvanometer coils became necessary in the author's recent experiments for determining resistance in absolute measure. He employs the method of Lord Rayleigh—a modification of Maxwell's arrangement of the bridge, with some alterations in regard to the application of current. Instead of noting the "throw" produced on a galvanometer by the current impulse of a single "make" or "break" at the battery circuit, the author, by the aid of a special current-interrupter, works with a succession of impulses of one kind, either "make" or "break," these producing a steady deflection of the galvanometer. By this means it is possible to test with much weaker currents than those necessary for the single impulse method; and thus the errors which arose from fluctuations of current, due to heating of the bridge-coils, with the larger currents of the older method, are no longer to be observed. Further, since the galvanometer-constant is eliminated from the final equation, the coefficient of self-induction is found quickly and conveniently, and with an accuracy, in the author's experiments, claimed to be ·05 per cent.

The coil to be investigated, w_1, is placed in branch I. of the bridge, the other branches being formed of inductionless resistances w_2, w_3, w_4; these are so adjusted that when a steady current is applied in the battery branch, the galvanometer branch has no current in it, that is—

$$\frac{w_1}{w_2} = \frac{w_4}{w_3}.$$

If the current is now broken, there arises in branch I. an E.M.F. $= P \cdot i_1$; and an induced current flows through the galvanometer, represented by

$$i_0 = P \cdot i_1 \frac{w_3 + w_4}{w_0(w_1 + w_2 + w_3 + w_4) + (w_1 + w_2)(w_3 + w_4)}$$

or

$$i_0 = PJ \frac{w_3}{w_0(w_1 + w_2 + w_3 + w_4) + (w_1 + w_2)(w_3 + w_4)} = P\frac{J}{S},$$

where P is the coefficient of self-induction to be determined; J the current in the battery branch; and

$$\frac{w_3}{w_0(w_1 + w_2 + w_3 + w_4) + (w_1 + w_2)(w_2 + w_4)} = \frac{1}{S}.$$

If the current J is made, and broken, n times a second, and the interrupter is so arranged that the bridge only receives the "break" impulses, that is, so that only the induction-currents due to the *opening* of the circuit can affect the galvanometer, the needle will take up a position a, for which the equation is

$$n i_0 = n \cdot P \cdot \frac{J}{S} = R \tan a, \quad \ldots \quad \ldots \quad (1)$$

in which R is the galvanometer-constant.

For the next operation the battery-circuit is permanently closed; and the resistance w_1, of the bridge-arm containing the coil to be investigated, is increased or diminished by an amount δ, and the needle is consequently deflected to an angle γ, in which case

$$j = \delta \cdot \frac{J}{S} = R \cdot \tan \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Provided that δ is small compared with the resistances in the bridge-arms, it follows from (1) and (2) that

$$P = \frac{\delta \tan a}{n \tan \gamma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To determine P it is only necessary, therefore, to observe the two angles a and γ with the same galvanometer; and to note the number of "breaks," n, per second, by means of the speed-counter of the interrupter. δ can be read off at a rheostat.

The most favourable conditions are obtained by choosing such a value for δ that $a = \gamma$. If P is very great, or in all circumstances where δ cannot be neglected as compared to w_1, w_2, w_3, w_4, the equations (2) and (3) take the form

$$j_1 = \delta \frac{J_1}{S_2} = R \tan \gamma; \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

$$P = \frac{\delta}{n} \cdot \frac{\tan a}{\tan \gamma} \cdot \frac{J_1}{J} \cdot \frac{S}{S_1}. \quad . \quad . \quad . \quad . \quad (3a)$$

It is shown that when δ is small compared to w_1, and when the resistance of the battery branch is given a favourable value, the fractions S/S_1 and J_1/J may each be made to approach unity. Numerical results are given for the particular coil which was used as the secondary bobbin in the author's determination of the ohm in absolute measure. Successive tests gave for $P \times 10^{-8}$ the values 3·0763, 3·0741, 3·0747 cm. respectively. The coefficient of self-induction of the same coil, calculated from the formula of Stefan, is $3·0721 \times 10^8$ cm. **R. A.**

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313. *Purification and Sterilisation of Drinking Water by Electrolysis.* **G. Oppermann.** (El. Rund. 12. pp. 42–43, 50–53, 1895.) —In the electrolysis of water, besides the chief products oxygen and hydrogen, small quantities of ozone and hydrogen dioxide are formed. The author describes a technical process for preparing drinking-water by the use of the ozone. The most suitable conditions for the production of ozone are : a temperature below 5° or 6°, small platinum electrodes, and high current-density. With these conditions the gaseous product contains about 6 per cent. of ozone. This ozone can be used to oxidise organic matter and convert nitrites and ammonia to nitrates. The excess of ozone causes the water to be unfit for drinking. In eight to fourteen days the water loses its unpleasant taste. To rapidly get rid of the ozone and hydrogen dioxide, the author proposes to submit it to a second

electrolysis with aluminium electrodes. These substances are then decomposed and aluminium hydroxide is formed. After filtration the water is fit to drink. Two forms of apparatus, one stationary and the other portable, are described, in which the operation may be performed.   S. S.

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314. *Electrochemical Detection of Boric Acid.* **M. Mayençon.** (J. Phys. élém. 10. p. 49, Jan. 1895.)—When a soluble borate is electrolysed, boric acid collects in the carbon anode. On removing the anode, the boric acid can be detected in the ordinary way.
                                                      S. R.

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315. *Electrochemistry of Fused Sulphides.* **J. Garnier.** (C. R. 120. pp. 184–185, 1895.)—Having previously shown (C. R. 1893, June 19) that an electric current of low pressure can be employed for the cementation of iron, the author has repeated his experiments, using metallic sulphides, and shows, from the analysis of the product, that the sulphur is eliminated from the fused sulphide. Using a Sudbury sulphide containing iron, copper, and nickel, he finds that the percentage of sulphur and copper diminishes from the anode to the kathode, whereas the percentage of iron and nickel increases in the same direction.   S. R.

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316. *Theory of Electrical and Optical Phenomena in connection with Moving Bodies.* **H. A. Lorentz.** (Leiden, 1895.)
[Continued from p. 134.]

[*Errata.* In the first part of this abstract, p. 129, line 5 from top, *for* div. **A** + *read* div. **A** = ; p. 130, lines 5, 7, 8 from foot, for v read **v** ; p. 132, last line, *for* $p \dfrac{\partial}{\partial y}$ *read* $p_y \dfrac{\partial}{\partial y}$.]

If we denote the current per unit area in a conductor by $\bar{\mathbf{S}}$, and define $\chi_x,\ \chi_y,\ \chi_z$ by the equations

$$\left\{ V^2 \nabla^2 - \left(p_x \frac{\partial}{\partial_x} + p_y \frac{\partial}{\partial_y} + p_z \frac{\partial}{\partial_z} \right)^2 \right\} \chi_x = \bar{\mathbf{S}}_x,\ \&c.,$$

it follows that

$$\bar{\mathbf{D}}_x = - \frac{1}{V^2} \left(p_x \frac{\partial}{\partial_x} + p_y \frac{\partial}{\partial_y} + p_z \frac{\partial}{\partial_z} \right) \chi_x,\ \&c.,$$

$$\bar{\mathbf{H}}_x = 4\pi \left(\frac{\partial \chi_y}{\partial z} - \frac{\partial \chi_z}{\partial \chi} \right),\ \&c.,$$

and the electric force on ions at rest (relatively to the moving ponderable matter),

$$\bar{\mathbf{F}}_x = - 4\pi \frac{\partial}{\partial_x} \left(p_x \chi_x + p_y \chi_y + p_z \chi_z \right),\ \&c.$$

The real electric force is, however, zero. For the force just found is given by a potential

$$\omega = - \left(p_x \chi_x + p_y \chi_y + p_z \chi_z \right) / (V^2 - p^2)$$

Let, then, for a moment the current be supposed not to exist, and an actual electric density of this amount be substituted in the conductor. The total change corresponding to this would be zero, and the electrification if left to itself would vanish. This may be expressed by saying that an electrification acting on the ions at rest sets them in motion, so that there is called forth another electrification of mean density $-\rho$. This electrification will be called forth in the actual case, and so also a force equal and opposite to $\overline{\mathbf{F}}_x$ will be brought into play.

As a particular case of this result it follows that a steady current in a wire carried by the earth exerts no electric force on another conducting circuit at rest relatively to the earth.

Another application of the methods here discussed is to prove that there is no force exerted by a charged body upon a conductor carrying a current.

We have now given a fairly full abstract of Lorentz's analysis; we shall in what follows sketch briefly his principal results, leaving the working-out to be supplied by the reader or obtained from the original paper.

First, we take induction in a linear conductor. Let a secondary B be displaced from rest in a configuration B_1 to rest in another B_2, while a primary A likewise passes from a configuration A_1 to another A_2; and let the primary current change from one constant value i_1 to another i_2. Let the time of transition be T. If the number of ions positively charged which pass any cross-section per unit of time be N, and the corresponding number of negative ions in the opposite direction be N', e and e' their respective charges, we have for the integral flow through the secondary

$$\int_0^T (Ne + N'e')dt.$$

If $\mathbf{F}_s$, $\mathbf{F}_s'$ be the forces on a positive and a negative ion in the direction of an element ds of the conductor, p and q be constant multipliers, ω the cross-section of the conductor,

$$N = p\overline{\mathbf{F}}_s\omega, \quad N' = q'\overline{\mathbf{F}}_s'\omega.$$

Let the velocity of the element of the conductor relatively to the moving axes of reference be v, that of the ions relatively to the wire w, then since w has the same direction as ds, $[\mathbf{w}\,.\,\mathbf{H}]_s = 0$, and

$$\mathbf{F}_s = \mathbf{F}_s' = 4\pi V^2 \mathbf{D} + [\mathbf{p}\,.\,\mathbf{H}]_s + [\mathbf{v}\,.\,\mathbf{H}]_s.$$

Thus the integral flow is

$$(pe + qe')\omega \int \{4\pi V^2 \overline{\mathbf{D}}_s + [\mathbf{p}\,.\,\mathbf{H}]_s + [\mathbf{v}\,.\,\mathbf{H}]_s\}dt,$$

mean values for the quantities depending on the convection current in a filament being denoted by bars above the symbols. This expression, if $\int ds/(pe + qe') = 1/C$, may be written

$$C\int \{4\pi V^2 \int \overline{\mathbf{D}}_s\,ds + \int [\mathbf{p}\,.\,\mathbf{H}]_s\,ds + \int [\mathbf{v}\,.\,\mathbf{H}]_s\,ds\}dt.$$

This formula may be made to give the fundamental law of induction, as follows :—Let $P = \int \bar{H}_n d\sigma$, where $\bar{H}$ is the normal component of the magnetic force, $d\sigma$ an element of a surface bounded by the linear conductor, and the integral is extended over the surface. Its value alters with the time from two causes, first the variation of H_n, second the variation of the field of integration. Thus in time dt the total change is

$$dP = dt \int \bar{H}_n \, d\sigma - dt \int [\mathbf{v} . \mathbf{H}]_s \, ds.$$

But this by the relations established may be written

$$-dt \int \{4\pi V^2 \bar{\mathbf{D}}_s + [\mathbf{p} . \mathbf{H}]_s\} ds - dt \int [\mathbf{v} . \mathbf{H}]_s \, ds.$$

The integral flow thus becomes

$$-C \int dP = C(P_1 - P_2),$$

P_1, P_2 being put for the values of the integral at the beginning and end of the time.

At any instant P of course depends on the value of $\mathbf{H}$ due to all causes. But since no induced current exists at the beginning or end of the time of transition, $\mathbf{H}$, and therefore P, then depends only on the current in the primary. The total flow for all the filaments, that is the total flow through the conductor, will then be

$$I = C_1(P_1 - P_2),$$

if C_1 be the sum of the values of C for all the filaments.

The part of the value of $\mathbf{H}$ depending on the motion of the earth is only of the order of p^2/V^2, and hence the law of induction is affected only to this extent by the earth's motion of translation relatively to the ether. This explains the failure of Des Coudres [Wied. Ann. Bd. 38. 1889] to obtain an effect of the order p/V produced by the motion of the earth on the induction of a primary coil placed midway between two secondaries. [The coils were arranged so that the common axis of the system was in the direction of the earth's motion, and the secondaries were joined in series and so adjusted that the inductive action on one was just balanced by that on the other. It was found that the balance was still maintained when the arrangement was turned through 180° about an axis at right angles to that of the coils. But it is easy to show that by the motion of the earth the front secondary is virtually placed nearer to the primary by the fraction p/V of the actual distance, and the other farther off from the secondary by the same amount. Thus apparently the balance should be disturbed if the earth and ether are in relative motion. No such effect was observed, which seems to show that the earth and the ether are relatively at rest, the result also apparently shown by the experiments of Michelson and Morley.]

The author now proceeds to consider the vibrations in the ether which are set up by ions the motions of which are oscillatory. In this case the quantities on the right-hand side of equations (A)

and (B) are known functions of x, y, z, t, periodic in t. If the period of oscillation of an ion be T, T will likewise be the period of the quantities $\mathbf{D}$, $\mathbf{H}$. For shortness $\mathbf{H}_x'$ is put for $\mathbf{H}_x - 4\pi(\mathbf{p}_y \mathbf{D}_z - \mathbf{p}_z \mathbf{D}_y)$, t' for $t - (p_x x + p_y y + p_z z)/\mathrm{V}^2$, and $\mathbf{H}_x'$ is regarded as a function of x, y, z, and t'.

We denote differentiation on this supposition by $(\partial/\partial x)'$, &c.; $\partial/\partial t'$; and put

$$(\nabla^2)' = (\partial^2/\partial x^2)' + (\partial^2/\partial y^2)' + (\partial^2/\partial z^2)'.$$

Then

$$\partial/\partial x = (\partial/\partial x)' - \mathbf{p}_x/\mathrm{V}^2, \quad \partial/\partial', \ \&c., \quad \partial/\partial t = \partial/\partial t'.$$

The equations for $\mathbf{H}'$ are now

$$\mathrm{V}^2 (\nabla^2)' \mathbf{H}_x' - \frac{\partial^2 \mathbf{H}_x'}{\partial t'^2} = 4\pi \mathrm{V}^2 \left[\left\{ \frac{\partial(\rho \mathbf{v}_y)}{\partial z} \right\}' - \left\{ \frac{\partial(\rho \mathbf{v}_z)}{\partial y} \right\}' \right], \ \&c.$$

If $\boldsymbol{\psi}_x$, $\boldsymbol{\psi}_y$, $\boldsymbol{\psi}_z$ be functions which fulfil the relations

$$\mathrm{V}^2 (\nabla^2)' \boldsymbol{\psi}_x - \frac{\partial^2 \psi_x}{\partial t'^2} = 4\pi \mathrm{V}^2 \rho \mathbf{v}_x, \ \&c.,$$

then

$$\mathbf{H}_x' = \left(\frac{\partial \psi_y}{\partial z} \right)' - \left(\frac{\partial \psi_z}{\partial y} \right)', \ \&c.,$$

&c., &c.

and if $\mathbf{H}_x'$ is found, $\mathbf{D}$, $\mathbf{F}$, $\mathbf{X}$ can be obtained from the equations already given.

It is to be noticed that t' can be regarded as the time reckoned from an epoch depending on the position of the point considered. If now we write $\rho \mathbf{v}_x = f(x, y, z, t')$, then it can be shown that

$$\psi_x = -\int \frac{1}{r} f\left(\xi, \eta, \zeta, t' - \frac{r}{\mathrm{V}} \right) d\tau, \ \&c.,$$

where x, y, z are the coordinates of a point for which ψ_x &c. are to be found, and ξ, η, ζ are the coordinates of any point at an element $d\tau$ of the space where $\rho \mathbf{v}_x$ &c. are not zero, r the distance between these two points. $f(\xi, \eta, \zeta, t' - r/\mathrm{V})$ is the value of $\rho \mathbf{v}_x$ at this latter point.

The waves excited in the medium by a single oscillating molecule are next considered. It is shown that if ϕ be the value of any quantity $\mathbf{H}_x'$, &c., $\mathbf{F}_x'$, &c. at P,

$$\phi = \mathrm{A} \cos \frac{2\pi}{\mathrm{T}} \left(t' - \frac{r}{\mathrm{V}} + \mathrm{B} \right),$$

where A and B are quantities depending on the length and position of the line $\mathrm{Q}_0 \mathrm{P}$ from the molecule to P, and r is the distance between these points. Let b_x, b_y, b_z be the direction-cosines of r, then

$$r = b_x(x - \zeta) + b_y(y - \eta) + b_z(z - \zeta)$$

and

$$t' = t - (\mathbf{p}_x x + \mathbf{p}_y y + \mathbf{p}_z z)/\mathrm{V}^2.$$

Substituting, we get

$$\phi = A \cos \frac{2\pi}{T} \left\{ t - \left(\frac{b_x}{V} + \frac{\mathbf{p}_x}{V^2} \right) x - \&c. + C \right\},$$

where

$$C = B + (b_x \xi + b_y \eta + b_z \zeta)/V.$$

Thus the direction-cosines of the normal to the wave-front are proportional to

$$b_x + \mathbf{p}_x/V, \quad b_y + \mathbf{p}_y/V, \quad b_z + \mathbf{p}_z/V,$$

and show the effect of the motion $(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$ upon the direction of the wave normal. If $\mathbf{p}_x = \mathbf{p}_y = \mathbf{p}_z = 0$, the wave normal is $Q_0 P$; in all other cases the normal is in the direction of the line joining P with the position which the source occupied when it excited the wave which is now at P.

After a discussion of Doppler's principle by means of these equations, the author considers the equations of motion of light for ponderable media. Calling M the value of $(\Sigma e\mathbf{q})/I$ [$e =$ charge, $\mathbf{q} =$ velocity of an ion, $I =$ volume of small sphere at point considered], he shows that the polarisation of the medium is $\bar{\mathbf{D}} + \mathbf{M}$. We denote this, which is Maxwell's displacement in the medium, and agrees with $\mathbf{D}$ in the free ether, by $\mathbf{D}'$. The equations become then, for the interior of the body,

$$\operatorname{div} \mathbf{D}' = 0, \quad \operatorname{div} \mathbf{H} = 0, \quad \operatorname{curl} \mathbf{H}' = 4\pi \dot{\mathbf{D}}', \quad \operatorname{curl} \mathbf{F} = -\dot{\mathbf{H}},$$

$$k_1 \mathbf{F}_x = 4\pi V^2 \mathbf{D}_x' + [\mathbf{p} \cdot \mathbf{H}]_x, \&c., \quad \mathbf{H}' = \mathbf{H} - [\mathbf{p} \cdot \mathbf{F}]/V^2,$$

where k_1, k_2, k_3 are electric constants of the medium for the directions x, y, z, and depending on the kind of light transmitted. These yield the previously obtained equations for the free ether.

At the surface of separation there is continuity of the normal components of polarisation, and the tangential components of $\mathbf{F}$ and $\mathbf{H}$.

The equation

$$\mathbf{F} = \sigma \mathbf{M} + j \operatorname{curl} \mathbf{M} + k [\dot{\mathbf{M}} \cdot \mathbf{p}],$$

in which σ, j, k are constants, is obtained for circularly polarising media. The last term of this corresponds to a turning of the plane of polarisation called forth by the motion of the earth.

The author proceeds next to the application of his results to optical phenomena, and modifies the equations of light propagation to suit the case of a system in motion, showing that for the quantities $\dot{\mathbf{D}}$, $\mathbf{F}$, $\mathbf{H}$ must be substituted other quantities which are the same functions of

$$x, y, z, t' \{ = t - (\mathbf{p}_x x + \mathbf{p}_y y + \mathbf{p}_z z)/V^2 \}$$

as these are of x, y, z, t. We can only state the applications and conclusions. The absolute and relative periods of a system are distinguished. The absolute period is that by which what goes on at a fixed point is reckoned, the relative is the same thing for a point which is in motion with the ponderable matter. Then it is concluded that :—

A state of vibration in a system in motion corresponds to one in a system at rest, when the relative period in the former is equal to the period in the latter.

In a system in motion relative rays of light of period T are reflected and refracted in the same manner as rays of the same period in a system at rest.

Interference and diffraction phenomena exactly correspond in the two cases.

Optical phenomena all take place just as if the earth were at rest, and the period that which it becomes according to Doppler's principle ; a result which includes the results obtained as to aberration by means of a telescope the tube of which was filled with water.

The path of the earth differs so little from a circle that the component of velocity along the radius vector can have but little effect on the apparent place of the sun as viewed from the earth. There is also no effect of the motion of the earth on the position of the lines of the spectrum whether produced by a prism or a grating.

Again, according to the theory, the motion of the earth is without any effect on the first order of magnitude upon the results of researches with terrestrial sources of light.

The effect of the motion of ponderable matter on the propagation of waves through it is next considered, with special reference to experiments which have been made by Fizeau and repeated by Michelson and Morley with improved apparatus. It is shown that according to the theory, if N denote the index of refraction of an isotropic body, *i. e.* the ratio V/W of the velocity of light in the ether to that in the body supposed at rest relatively to the ether, W' the velocity of light in the direction of the wave normal in the body supposed moving with velocity p_n in that direction, then

$$W' = W - p_n/N^2.$$

Also, if W'' be the velocity of the light relatively to the ether,

$$W'' = W \pm \left(1 - \frac{1}{N^2}\right) p_n,$$

which agrees with Fresnel's assumption. If n be the true index of refraction, that is for the body at rest, the value of W'' may be written

$$W' = \frac{V}{n} \pm p\left(1 - \frac{1}{n^2}\right) \mp \frac{p}{n} T \frac{dn}{dT}.$$

The equation has been verified by the experiments of Fizeau and of Michelson and Morley on the velocity of light in flowing water, at least so far as the first two terms on the right are concerned. The author thinks it not impossible to detect the influence of the last term, and thus to observe the Doppler alteration of the period produced by the motion.

The author next discusses double refraction, and comes to the conclusion that double refraction is independent of the earth's

motion; a result easily obtained by considering the motion along the ray instead of that along the wave normal.

Passing over a discussion of some questions of energy and rotation of the plane of polarisation in quartz as influenced by the motion of the earth, we come to a consideration of the bearing of the theory on the results of Michelson and Morley's experiments on the interference of two rays travelling over as nearly as possible equal paths, but one in the direction of the earth's motion, the other at right angles to that direction. As is well known, the experiment which, if the earth were in motion relatively to the ether, should have shown a difference in the positions of the interference bands when the whole apparatus was turned through 90° round a vertical axis, gave a negative result. Thus, apparently, the earth does not move relatively to the ether at its surface, at least within the walls of a building. This result is at variance with the theory of aberration. Lorentz recalls a possible explanation which he and Fitzgerald have put forward independently, that the length of a beam or rod of ponderable matter is slightly different when the rod is placed in the direction of the motion of the earth, from what it is when the rod is at right angles to that direction. In fact the result would be explained if the length L were increased by $Lp^2/2V^2$ when turned from the former direction to the latter.

The author then points out that we are led to exactly this alteration of dimensions if we assume that the law stated above for the electrostatic forces in the moving system holds for the molecular forces. For consider two systems of S_1, S_2 of molecules, the first moving in the direction of the x axis with velocity p, the second at rest, between whose dimensions the relation stated above holds, and assume that in both systems the x components of force are the same, while the y and z components differ by the factors $\sqrt{1-p^2/V^2}$, as stated above. It is clear that the forces in S_1 and S_2 vanish together. Accordingly, if S_2 is in equilibrium, the molecules in S_1 have exactly those positions in which they can remain in relative equilibrium in the body in motion. The motion would thus, if impressed on a body, give rise to the alteration of dimensions specified, namely to a shortening in the ratio 1 to $\sqrt{1-p^2/V^2}$, that is by the fraction $p^2/2V^2$ of the original length.

Of course the identification of the law of molecular forces with that of the electrostatic forces requires justification. The molecules are not at rest, but are really in a " stationary state," in which, as is supposed in so many other cases, the results deduced on the supposition that the body may be regarded as of parts at rest are assumed to hold.

The essay concludes with a discussion, according to the theory of the author, of Fizeau's experiments on the turning of the plane of polarisation of sunlight produced by oblique passage through glass, (1) with the polariser towards the east and the analyser

towards the west, (2) with the arrangement reversed. Fizeau found a constant difference between the readings obtained for these two positions of the apparatus. This result is not in accordance with the theory, which requires that there should be no effect. The author suggests that possibly some difference between the results in the two cases might arise from the different positions of the heliostat mirror, which rendered the angle of incidence different in the two cases ; but against this is to be placed the fact that it was found by Fizeau that replacing the metallic mirror of the heliostat by a totally reflecting prism did not affect the result.   A. Gr.

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317. *Resistance of some new Alloys.* **E. van Aubel.** (J. Phys. 4. pp. 72–74, Feb. 1895.)—Kruppine is the name of an alloy made at the Krupp steel-works. In the first set of measurements a wire, after heating for several hours at 150° C. and then for two hours at 200° C., gave a specific resistance at 18° C. of 84·7 microhms per $\frac{cm}{cm^2}$. Its mean temperature coefficient between 18° and 50° was +0·000791. Its specific gravity was 8·107. In the second set, after heating for several days at 170° C., the specific resistance in the same units was 85·5, and the mean temperature coefficient, between 25° and 73°, 0·00076. A comparison with other materials shows that the alloy has a greater specific resistance, and only falls short by 10 per cent. from that of mercury. This alloy may be heated to 600° C. without changing its structure. Three nickel alloys, after heating for twenty-four hours at 130° C., gave results which showed high specific resistance combined with a small temperature coefficient. One of the alloys has a greater specific resistance and smaller temperature coefficient than the manganin which is actually used at Charlottenburg for the construction of standards of resistance.   S. S.

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318. *Electromotive Force of Magnetisation.* — **Hurmuzescu.** (Bull. Soc. Fr. Phys. 56. 1895.)—There is a potential difference between two similar pieces of metal dipped in an electrolyte, if one of them be magnetised. This the author verified, with the capillary electrometer (no measurements given). In agreement with theory, iron and nickel become positive by magnetisation, bismuth negative. According to M. Janet, the differences may amount to $\frac{1}{100}$ volt.   R. A. L.

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319. *Conduction of Heat and Temperature in Geissler Tubes.* **E. Warburg.** (Wied. Ann. 54. pp. 265–275, 1895.)—Assuming all the electrical energy of the current in a Geissler tube is converted into heat, the author, by the usual methods used in the conduction of heat, finds expressions for the temperature along the axis of the tube, the temperature of the inner wall, and the

mean temperature of the gas, when a stationary state has been reached. Using the results of A. Herz's experiments on the potential gradient in nitrogen, he finds that if the external medium is at $0°$, the strongest current used by Herz ($\cdot 0032$) would not give a temperature of more than $133°$ at the axis, the pressure of the gas being 8 mm. With the same current and pressure the mean temperature of the gas is $69°\cdot 6$, and the temperature of the inner wall $1°\cdot 29$. With hydrogen at 8 mm. and a current $\cdot 001$, the axial temperature is $6°$. The energy expended in radiation is assumed to be small, and is neglected. Since a rise of temperature of several hundred degrees has no effect on the brightness with which a gas glows, the author concludes that the glow is a direct effect of the current, and is not due to the temperature. According to Hittorf the potential gradient is independent of the current if the density is constant. In Herz's experiments it diminished with increasing current. The author ascribes this to diminished density due to rise of temperature. Assuming the pressure constant, he calculates the potential gradient for different strengths of current and obtains values of the same order of magnitude as those observed by Herz.

Using the usual equations of the variable state, he finds the time taken to reach the stationary state by nitrogen at 10 mm. pressure in the tubes used by Herz is from $\cdot 014$ to $\cdot 33$ second.

<div align="right">J. W. C.</div>

320. *Action of Kathode Rays on Salts.* **E. Goldstein.** (Wied. Ann. 54. pp. 371–380, 1895.)—Lithium chloride, under the action of the kathode rays, phosphoresces bright blue. After a time the phosphorescence grows weaker, and the salt is found to have taken a heliotrope or violet colour, which remains after the electric discharge is stopped. In a vacuum or in dry air the colour remains, in moist air it disappears in a few seconds. If the coloured salt is heated moderately, it becomes flesh-coloured or brown according to the depth of its original colour ; if it is heated very strongly it becomes colourless. The colour cycle can then be repeated. With differences in the details, similar phenomena are shown by sodium chloride, potassium chloride, potassium bromide, potassium iodide, potassium carbonate, sodium bromide, lithium bromide, and lithium iodide. Salts of the alkaline earths show no colour. Some of these salts, like lithium chloride, lose their colour very quickly when exposed to the open air ; others, as potassium chloride, retain it for as long as twenty-four hours. Contact with water in every case destroys the colour at once. Sealed up in dry air or in a vacuum, lithium chloride retains its colour after two months. The others lose their colour more or less completely after a week or two whether kept in the dark or not.

The author convinced himself by special experiments that the colour is not due to impurity or to decomposition, and is of opinion that it is due to the formation of a physical isomer.   J. W. C.

321. *Distribution of Induced Magnetism.* **M. Ascoli.** (N. Cim. 4. 1, pp. 5–18, Jan. 1895, & 4. 1, pp. 108–127, Feb. 1895.) —In this paper the author has collected together the substance of a number of separate papers on the subject of induced magnetism which he has published in various periodicals. Experiments are described for determining the distribution of the induced magnetism in solid and hollow iron cylinders when placed inside a magnetising helix through which a current of known intensity is passed. In most cases the cylinders experimented on are formed of a number of soft iron wires arranged in regular layers. Coils of fine insulated copper wire are wound between the layers of iron wires, and serve to measure the induction through the cylinders at different distances from the axis. Cylinders having lengths varying from 54·6 to 5·6 times the diameter are examined, and the induction at different distances from the axis and for different strengths of the inducing-field are given in a series of tables and curves. From these the author comes to the conclusion that the radial distribution is practically uniform for long cylinders, *i. e.* those in which the length is greater than thirty-six times the diameter. In shorter cylinders there is an increase in the intensity of the induced magnetisation towards the surface of the cylinder, which becomes more marked as the length of the cylinder is decreased. While in short cylinders this increase is most marked for strong inducing-fields, in the case of long cylinders any small increase that occurs seems most marked for weak fields. The longitudinal distribution is examined by having a number of secondary coils wound at different distances from the end of the cylinders under examination. The ratio of the intensity of magnetisation near the axis to that near the surface is found to be constant along almost the whole length of the cylinder; it is only quite towards the ends that the intensity of magnetisation of the iron near the axis decreases more rapidly than that of the iron outside. W. W

322. *Electrolytic Conductivity Curves.* **F. Swarts.** (Zschr. phys. Chem. 16. pp. 118–124, 1895.)—Ostwald has experimentally investigated the connexion between the conductivity of solutions of organic acids and their molecular volumes. For certain acids he found that the conductivity-curves had a critical point, at which the direction of their curvature changed. The author investigates mathematically the position of this critical point from the equation

$$\frac{\mu_\infty (\mu_\infty - \mu_v) v}{\mu_v^2} = c,$$

where μ_v and μ_∞ represent the molecular conductivities when the volume of a given mass of acid is v and ∞ respectively. He finds, as an approximate relation, that at the critical point

$$v = \frac{2}{3} c.$$

W. C. D. W.

· 323. *Thermodynamics of Galvanic Polarisation.* **H. Jahn** and **O. Schönrock.** (Zschr. phys. Chem. 16. pp. 45–71, 1895.)— When the kation of an electrolyte is absorbed by the platinum electrode, so that a solid solution of ions is formed, the change in free energy during the process is given by

$$\delta U - TS,$$

where U is the total energy, S the entropy, and T the absolute temperature. From the formula given by Planck, the author deduces as an expression for the total kathode polarisation

$$\pi = \Sigma \delta U - T\,(S_m - \log C_m - \sigma_1 + \log c_1)\,\delta n,$$

where S_m denotes the entropy of one of the molecules dissolved in the solid electrode, C_m their concentration, σ_1, C_l the corresponding quantities for the ions in the liquid, and n the number of ions passing into the electrode when unit quantity of electricity flows round the circuit. By considering the platinum ions which pass from the electrode into the liquid, and framing a similar equation for the anode polarisation, an expression for the total polarisation is obtained. Since the concentration of gaseous molecules in the liquid increases with the pressure, it follows from the equation that the polarisation at which the evolution of gas begins must increase as the pressure rises. In dilute sulphuric acid at a pressure of 10 mm. of mercury, gas is evolved when the polarisation is 1·63 volt, and at 742 mm. when it reaches 1·79–1·82 volt. The polarisation for gas evolution should be the same for all oxyacids; and Le Blanc found :—Sulphuric acid, 1·67; nitric acid, 1·69; phosphoric acid, 1·70; monochloracetic acid, 1·72; malonic acid, 1·69. In haloid acids the process is complicated by the decomposition of water by chlorine &c. The polarisation at which gas begins to be evolved rises from 1·26 volt for $\frac{2}{1}$ normal solutions of hydrochloric acid to 1·69 for $\frac{1}{22}$ normal solutions.

An experimental investigation of the total polarisation of platinum electrodes in dilute sulphuric acid was made, a high-resistance galvanometer being used to measure it. Calling the maximum polarisation p_0, and the corresponding current in the primary circuit J_0, the authors show that the value of

$$p_0 - \gamma \log J_0,$$

where γ is a constant, should always keep the same at the same temperature when the primary current, or the concentration of the electrolyte is altered. The results of these observations confirm this deduction. The maximum polarisation should be independent of the size of the electrodes, but since small ones are more quickly saturated than large ones, they usually give greater values. In a dilute solution of sodium chloride, the polarisation is greater than in acids by reason of the alteration in free energy due to the decomposition of water by the sodium at the kathode. Similar equations, for such a case as caustic soda, show that the total

polarisation must be the same as for dilute oxyacids, and the same for weak and strong bases. Observations by Le Blanc confirm this, for he gets :—Soda, 1·69 ; potash, 1·67 ; ammonia, 1·74 ; ethylamine, 1·75 ; diethylamine, 1·68. W. C. D. W.

324. *Electrolytic Corrosion of Underground Metal-work*. **G. P. Low.** (Trans. Am. Inst. El. En. 12. pp. 113–128, 1895.)—The paper is a discussion of the causes of electrolytic corrosion of water-mains &c. due to earth-return currents of electric railway circuits, and an investigation of means to reduce such electrolytic effects as much as possible. The author concludes that corrosive effects may be eliminated by judicious bonding, and lays down the following working rules :—(1) There should be no cross-bonding at points where the mains show permanently negative potentials ; (2) The cross-bonding should be heavy at points where the mains show permanently positive potentials : (3) Rail-bonds should be permanent and reliable ; (4) The track return circuit should have high conductivity ; (5) Where the mains show permanently negative potentials the resistance between track and mains should be high ; (6) Generators should permanently feed positive to line and negative to track and main ; (7) There should be heavy bonding between the negative side of the generators to various points of the mains. In order to reduce the resistance of the track return circuit, the author advocates the use of direct bonding, and points out the advantages attending it. W. G. R.

325. *Alternating-Current Method of determining the Resistance of Electrolytes*. **F. Kohlrausch.** (Zschr. phys. Chem. 15. pp. 126–130, 1894.)—A reply to a paper by Wildermann raising objections to the telephone method of measuring the resistance of badly-conducting liquids. The author ascribes the difficulty in obtaining a sharp minimum of sound mainly to the electrostatic capacity of the cell and the resistance-coils. If the cell is in a bath, the latter should contain a non-conducting liquid such as petroleum, but it is better not to use a bath. The detrimental effect of the capacity can be obviated by placing a small adjustable condenser in multiple arc with the comparison resistance. Details are given of the construction of suitable condensers, and several other minor points affecting the accuracy of the method are discussed. J. W. C.

326. *Electrostatic Capacity of Coils*. **J. Cauro.** (C. R. 120. pp. 308–311, 1895.)—Double-wound coils have negligible self-induction, but possess a capacity which increases rapidly with the resistance. The ordinary method of measuring self-induction measures the algebraic sum of the self-induction and capacity, and since capacity is equivalent to a negative self-induction the result of such a measurement may be either positive or negative : if

positive the algebraic sum may be made zero by the introduction of a suitable amount of paraffin to increase the capacity ; if negative, the self-induction may be increased by the introduction of a suitable soft-iron core. The capacity may be considerable, and must be eliminated whenever the elimination of self-induction is considered necessary. The author states that, in his attempts to reduce the combined effect of self-induction and capacity in resistance-coils, he has obtained excellent results by starting the winding of alternate layers of wire from the same end of the bobbin. W. G. R.

327. *Magnetic Shielding.* **M. Ascoli.** (Wied. Ann. 54. pp. 381–384, 1895.)—This is a criticism of some conclusions of Hr. O. Grotrian, with regard to the distribution of magnetism within hollow and solid cylinders respectively. No experimental details are given, but the result is stated that the magnetisation of a short, hollow cylinder is only very slightly increased if an iron core is inserted within it while in the magnetising field. Grotrian considers the screening action of the outer tube upon the core. Ascoli objects to the idea of " screening " in this sense, and thinks that, according to the theory of magnetic circuits, the effect is more properly explained as being the result of the different specific magnetic resistances of the cylinder and core respectively. While admitting that a sheet of iron acts as a screen towards a non-magnetic medium, or towards small magnetic bodies placed behind it in such a medium, he thinks that it is a misuse of terms to speak of a sheet as " screening " a medium having a permeability similar to its own. The case does not seem to be fully made out ; for it is not laid down whether " screening " is to be regarded as the action of a permeable magnetic jacket, or of an impermeable bounding medium ; or both. R. A.

328. *Kathode Polarisation.* **J. Roszkowski.** (Zschr. phys. Chem. 15. pp. 267–322, 1894.)—Owing to the rapid fall in polarisation when the primary current is broken, the author used the method due to Fuchs, in which measurements of the fall of potential are made while the primary current is flowing. A tapping electrode, described by Ostwald, was employed. It consisted of mercury, covered by a layer of calomel, placed beneath a normal solution of potassium chloride in a bottle. A siphon-tube, also filled with the potassium chloride solution, served to make contact with the liquid in the polarising cell. The polarisation reached a maximum in less than three minutes after the primary circuit was closed. The influence of increasing the primary current was very marked. With platinised platinum electrodes, the kathode polarisation varied from $+0.230$ volt to -0.180 volt when the resistance outside the thermopile which supplied the primary current was increased from 0 to 800 ohms, above which resistance it remained constant. With mercury kathodes 1 sq. cm. in area and

small currents, the H-polarisation was algebraically greater than
with a kathode of 2 sq. cm. area, whether the primary current
was adjusted to give it a positive or a negative value, but
when the primary E.M.F. reached 3·3 volts the polarisation
seemed to be independent of the size of the electrodes. It was
more regular with a platinised platinum electrode than with any
other kind of surface, and was then proportional to the E.M.F. of
the primary current, while the E.M.F. varied from 1·5 to 3·4
volts. With mercury and various amalgams for kathode, too, no
maximum of polarisation was reached as the primary E.M.F. was
increased. A similar result was obtained with Wood's metal, and
no sudden change in polarisation was observed as it was heated
past its melting-point. Liquid electrodes gave a nearly constant
value for the kathode polarisation, independent of their nature.

W. C. D. W.

CHEMICAL PHYSICS.

329. *Double Decomposition of Gases.* **H. Arctowski.** (Zschr.
an. Chem. pp. 213–223, 1895.)—The author has studied the
reaction of sulphuretted hydrogen on volatile metallic chlorides
at a temperature at which the metallic sulphide produced is the
only possible solid.   _____   .   S. R.

330. *Thermochemistry of Solution.* **E. Monnet.** (C. R. 120.
pp. 500–501, 1895.)—The author has investigated the variation in
the heat of dissolution of acetate of soda in water for different
concentrations, and finds that it increases with the percentage of
salt, so that heat is evolved on the dilution of strong solutions.
          _____ ___   S. R.

331. *Resistance to Corrosion of Aluminium Alloys.* **J. W.
Richards.** (Journal Frankl. Inst. pp. 69–72, 1895.)—This paper
is only of commercial interest, and deals of the action of acids,
alkalis, common salt, and carbonic acid gas on aluminium and its
light alloys.   The author's experiments with nitric acid show that
the purest aluminium is very suitable as a substitute for platinum
in Groves' battery, and that a sheet of 99·9 per cent. metal, one
tenth inch in thickness, would bear over a month's constant
immersion in the concentrated acid.   .   S. R.

332. *Influence of Hydration on Solubility.* **N. Kurnakow.**
(Zschr. an. Chem. 8. pp. 103–114, 1895.)—In general the solubility
of lower hydrates, and of an anhydrous salt, is greater than that of
higher hydrates.   The author, however, points out that with many
compound metal bases the reverse is the case.   Thus the halogen
salts and nitrate of the roseopentamine compounds of cobalt,
chromium, iridium, and rhodium were more soluble as the hydrate,
$MX_3.5NH_3.H_2O$ (where M = Co, Cr, Ir, Ro), than as the
anhydrous purpureo-salt, $MX_3.5NH_3$; and similar results are
obtained in the case of other ammonia compounds of the general
type $MX_3.nNH_3$.   Among inorganic compounds the same thing
holds good for many sulphates, such as those of chromium, iron,
cobalt, nickel, zinc, and magnesium, and for the halogen salts of
chromium and the platinum group of metals.   An experimental
investigation showed that the solubility and the temperature-
coefficient of solubility of $CoCl_3.5NH_3.H_2O$, $CoCl_3.6NH_3$, and
$CoCl_3.5NH_3$ were in the order in which the compounds are
named.   _____   W. C. D. W.

333. *Molecular Weight Determination.* **E. Beckmann.** (Zschr.
phys. Chem. 15. pp. 656–680, 1894.)—A review of the various
forms of apparatus for applying Raoult's method of determining
molecular weights by means of the boiling-point of a solution. S. S.

334. *Critical State.* **K. Wesendonck.** (Zschr. phys. Chem. 15. pp. 262–266, 1894.)—The author wishes to decide which of the phenomena observed in the neighbourhood of the critical temperature may be regarded as indicating the actual critical point. No new experiments are described. S. S.

335. *Electromagnetic Rotation of Solutions of Hydrochloric Acid.* **O. Schönrock.** (Zschr. phys. Chem. 16. pp. 29–44, 1895.)— Perkin called in question some of the results of the author. He therefore repeats the experiments, and adds to them others, and shews that both Perkin's and his work is correct. He also examines the conclusion of Ostwald that ionisation has a direct influence on electromagnetic rotation. The method of making the measurements remains the same as in his inaugural dissertation. The following table gives a summary of the results, ρ being the percentage of dissolved substance and S the molecular rotation :—

Substance.	ρ.	S.	Observer.
	per cent.		
HCl in Water	11·45	4·774	Schönrock.
	15·63	4·419	Perkin.
	25·06	4·404	Schönrock.
	25·60	4·405	Perkin.
	30·86	4·303	,,
	36·50	4·215	,,
	41·70	4·045	,,
HCl in Amyl Ether ...	0·76	4·036	Schönrock.
	2·34	2·838	,,
	3 45	3·322	,,
	8·96	2·101	,,
	10·68	2·211	Perkin.
	12·82	2·265	,,
	14·41	2·261	,,
HCl in Amyl Alcohol.	1·58	5·413	Schönrock.
	6·89	4·028	,,
	9·49	3·809	,,
	25·45	3·285	Perkin.
	28·03	3 288	,,
HCl in Ethyl Alcohol.	6·59	4·491	Schönrock.
	9·78	4·201	,,
	14·64	4·232	,,
	40·04	3·365	Perkin.

From this table it appears that ionisation has no influence on the rotation, the numbers not being proportional to the corresponding ionisations. It is, however, clear that with increasing concentration the rotatory power falls, whilst, as Perkin has shown, in the case of sulphuric and nitric acids and their salts there is an increase in rotation with increase of concentration. So that the electromagnetic rotativity is not independent of the concentration. S. S.

336. *Freezing-Points of Dilute Solutions.* **W. Nernst** and **R. Abegg.** (Zschr. phys. Chem. 15. pp. 681–693, 1894.)— Different determinations of freezing-points have given results which differ among themselves more than can be accounted for by experimental errors. The authors therefore consider in detail the process of freezing a solvent. If T_0 is the true freezing-point of a large mass of liquid, and t its actual temperature at a time z, the alteration of temperature in a time dz is

$$dt = K(T_0 - t)dz.$$

In a limited volume of liquid cooled by radiation to an outer enclosure, heat equilibrium would be reached at a temperature t_0, which in general differs from T_0. The change of temperature is given by

$$dt = k(t_0 - t)dz\,;$$

so that the actual alteration is

$$dt = [K(T_0 - t) + k(t_0 - t)]dz,$$

which on integration gives the observed freezing-point, *i. e.* the temperature at which the thermometer becomes stationary:

$$t' = T_0 - \frac{k}{K}(t' - t_0).$$

This only coincides with the true freezing-point when either t_0 is equal to T_0 or K is infinite. Heat is developed by the process of stirring, but if the speed of the stirrer is kept constant, allowance can be made for this. Measurements were made with a Beckmann thermometer. The solution was placed in a beaker, which was separated by an air-space about 1 cm. in thickness from an outer vessel containing a cryohydrate of known freezing-point. The constants for the apparatus were determined by measuring the rate of cooling with known differences of temperature. The following table gives the corrected values for the molecular depression of the freezing-point of solutions of cane-sugar, m denoting the number of gram-molecules to 1000 g. water and m' the number of gram-molecules per litre of solution.

$m.$	Molecular depression.	$m'.$	Molecular depression.
0·0178	1·88	0·01785	1·89
0·03534	1·79	0·03505	1·81
0·0688	1·81	0·06776	1·84
0·1305	1·88	0·1269	1·93

The results of other observers are compared. For a 1-per-cent. sugar solution the authors find a molecular depression of 1·86; Arrhenius gives 2·02, Raoult 2·07, Jones 2·18, and Loomis 1·81.

W. C. D. W.

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337. *Freezing-points of Dilute Solutions.* (3 Papers.) **M. Wildermann** and **P. B. Lewis.** (Zschr. phys. Chem. 15. pp. 337–385, 1894.)—The first-named author finds that in determining the freezing-point of pure water an ice-cap is liable

to form on the thermometer bulb. When this is prevented, the freezing-point rises by 15 to 17 ten-thousandths of a degree. He uses a large volume of liquid—about 1250 c. c.—and keeps it very thoroughly stirred. Mercury thermometers are used, one graduated to 1/100 and another to 1/1000 of a degree. Van't Hoff's formula,

$$t = \frac{0 \cdot 02 \ T^2}{W} = 1 \cdot 87,$$

in which t is the molecular depression of the absolute freezing-point T, and W the latent heat (80), is then confirmed for dilute aqueous solutions of cane-sugar, urea, and alcohol, the agreement being good. In the case of alcohol the result is about $1\frac{1}{2}$ per cent. too low, but this is probably due to errors in estimating the concentration. The author then investigates Arrhenius' dissociation hypothesis, and shows that the degree of dissociation which must exist in order to explain, by this means, the depression of the freezing-point in dilute aqueous solutions of sulphuric acid, potassium chloride, dichloracetic acid, trichloracetic acid, and nitrobenzoic acid is about the same as that calculated from the electrical conductivities. The validity of Ostwald's law of dilution,

$$\frac{\alpha^2}{(1-\alpha)v} = \text{constant,}$$

where α is the ratio of the actual to the limiting molecular conductivity, and v the volume of solution containing one gram-equivalent of dissolved substance, was then confirmed by observations on the freezing-points of solutions of dichloracetic and nitrobenzoic acids.

In the next paper the author gives details of his determination of the freezing-point of pure water on his thermometers, and considers the effect of the formation of an ice-cap and other disturbing conditions.

The third paper is an account by the late P. B. Lewis of the apparatus and method used, with details of the calibration of the two mercury thermometers.   W. C. D. W.

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338. *Freezing-Point of Solutions of Sodium Chloride.* **A. Ponsot.** (C. R. 120. pp. 317–319, 1895.)—In a previous paper the author described a new method of determining the freezing-point of aqueous solutions. He has applied it to NaCl, KCl, and KBr, and now gives the results for NaCl. He compares his results with those of Jones, Arrhenius, Loomis, and Pickering, and finds that they are quite different. He considers that these observers have not determined the true freezing-point when there is equilibrium between the ice and the solution, but only the point when the heat yielded to the thermometer by the formation of ice is not greater than that which it receives by radiation. He does not admit the conclusion about ionisation or the formation of hydrates.

His results indicate that the lowering is proportional to the weight of salt dissolved in 100 grams of solution, and he intends to show that this is in agreement with the laws of osmotic pressure.　　　　　　　　　　　　　　　　　　　　　　　　S. S.

339. *Lowering of Freezing-point, and Vapour-Pressure of Dilute Solutions.* **A. Ponsot.** (C. R. 120. pp. 434–436, 1895.)—By means of an equation of van't Hoff's, and of an equation already published by the author (C. R. Oct. 1894) expressions are deduced for the lowering of the freezing-point and for the diminution of vapour-pressure.　　　　　　　　　　　　　　　　　　　　S. S.

340. *Lowering of Freezing-Point of very Dilute Solutions.* **A. Leduc.** (C. R. 120. pp. 436–439, 1895.)—The great differences between the results of different observers for the lowering of the freezing-point in dilute solution lead the author to propose the substitution of a method depending on observations of the pressure required to lower the freezing-point of water to that of the solution. It would be necessary to see whether a crystal of ice would grow or diminish when introduced into the solution under pressure. This should be a sensitive method, since atmospheric pressure only depresses the freezing-point by $0°·0076$.

If U is the specific volume of the solvent in the solid state, and U′ in the liquid state, and w the osmotic pressure of the solution, then the change of pressure will be $w \dfrac{U'}{U - U'}$.　　　　　　S. S.

341. *Cryosalts (Cryohydrates).* **A. Ponsot.** (J. Phys. 4. pp. 67–71, Feb. 1895.)—The temperature at which a cryohydrate is formed is given by the intersection of two lines. The ordinates of one of these lines represent at each temperature (abscissæ) the maximum concentration of the solution when solid salt is present. The ordinates of the other line represent at each temperature the minimum concentration of the solution in presence of solid solvent. Beyond the point of intersection of these lines the solution is at once supersaturated with salt and solvent, so that when solidification takes place both of these solids separate. This mixture was called by Guthrie the cryohydrate of the salt. The author proposes to substitute the term cryosalt. Taking a solution of salt of exactly the cryohydrate strength at the temperature of fusion of the cryohydrate, the following experiments show that it is a mixture. Addition of a crystal of ice produces further crystallisation of pure ice. Addition of crystals of salt produces the formation of more crystals of salt. Evaporation of the solution at the temperature of fusion produces crystals of salts. Microscopic examination with coloured salts, and with the polariscope for colourless salts, shows that the solid formed is a mixture of crystals. By repeated fusion and resolidification of the cryohydrate it is possible to separate the water from the salt.　　S. S.

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